

Monetary Policy Trade-offs in a Low Carbon Transition Scenario

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Abstract

This paper studies the role of monetary policy during the low-carbon transition in an economy with carbon taxation, financial frictions, and endogenous abatement. I develop a two-sector DSGE model with green and brown capital, banking constraints, and Epstein-Zin preferences to examine how transition risks propagate through asset prices and balance sheets. Carbon taxes operate as supply-side shocks that generate asset stranding and weaken bank net worth, amplifying real effects through the financial accelerator. I evaluate standard and emissions-augmented Taylor rules under welfare maximization and stabilization objectives. While aggressive accommodation of declining emissions can improve welfare by smoothing consumption risk and supporting investment, it does so at the cost of substantial inflation volatility, rendering such policies incompatible with conventional central bank mandates – a structural tension between household welfare and price stability objectives. In contrast, a small emissions response improves stabilization outcomes without materially affecting the emissions path, which remains determined by fiscal policy. The results support a pragmatic role for monetary policy that smooths financial adjustment during the transition while preserving price stability.

1 Introduction

Climate change is an urgent global challenge that requires immediate action from policymakers, economists, and stakeholders. Within macroeconomics and finance, transition risks have gained prominence as economies shift toward a low-carbon future. These risks stem from the structural transformations from climate policies, technological advances, and evolving consumer preferences (Feyen et al., 2020). As governments implement increasingly ambitious climate policies to meet Paris Agreement commitments, central banks confront a fundamental question: how should monetary policy respond to carbon pricing shocks, and does the answer depend on how these policies are implemented?

The transition to a low-carbon economy presents significant risks of economic and financial instability, particularly for carbon-intensive sectors that rely heavily on fossil fuels. Industries such as oil, gas, coal, steel, aluminum, cement, and greenhouse horticulture are especially vulnerable to asset devaluations, potentially leading to stranded assets and severe financial repercussions (Van der Ploeg and Rezai, 2020). These disruptions could trigger rising debt defaults, speculative bubbles in emerging industries, and abrupt asset revaluations, creating systemic risks that extend across the broader economy (Semieniuk et al., 2021). Financial feedback effects can further amplify macroeconomic impacts, with expectations playing a critical role. As capital is reallocated, carbon-intensive firms may experience declining profitability, asset devaluation, and tighter credit conditions, while low-carbon firms benefit from improved financing (NGFS, 2024).

Climate policy and monetary policy are interconnected. Carbon taxes operate as supply shocks that raise production costs in emission-intensive sectors, creating inflationary pressures while simultaneously depressing output, a classic policy trade-off for central banks. Schnabel (2022) has acknowledged that climate change could force it to take ‘Greenflation’ into account in monetary policy operations, while Diluio et al. (2021) emphasizes that mitigation policies, like carbon pricing, can be a direct source of shocks, creating potential trade-offs for monetary policy. Yet despite this growing policy attention, the academic literature provides limited guidance on how central banks should optimally calibrate their interest rate response to carbon tax transitions, and whether this response should depend on how climate policies are communicated and implemented.

According to the Intergovernmental Panel on Climate Change (Lee et al., 2023), there is a sig-

nificant gap between the emissions reductions achieved under current policies and those required to meet the Paris Agreement targets, prompting the need for additional policy action by governments. From a macroeconomic perspective, beyond the direct impacts of climate mitigation policies, this policy gap introduces uncertainty about the transition pathway, which can affect investment decisions and long-term economic planning. To effectively manage these transition risks, it is essential to integrate them into macroeconomic models that account for financial frictions and environmental externalities.

In this context, I employ a two-sector Environmental Dynamic Stochastic General Equilibrium (E-DSGE) model with financial frictions, building on Gertler and Karadi (2011). The model incorporates Epstein-Zin preferences, and environmental externalities through a brown sector that emits pollutants and a green sector that operates sustainably. Firms in both sectors issue bonds to finance capital, with banks holding these bonds under leverage constraints, creating a spread between bond yields and deposit rates. The model includes two features that prove central to the results: endogenous abatement technology that allows brown firms to reduce emissions in response to carbon pricing, and financial frictions in the banking sector that can amplify macroeconomic fluctuations during the transition.

I characterize optimal policy by searching the parameter space of Taylor-type rules. The baseline specification is the standard Taylor rule with responses to inflation (ϕ_π) and output (ϕ_y). I then consider, normatively, an *emissions-augmented* Taylor rule that adds a response to emissions growth (ϕ_e), following proposals in the literature (Chen et al., 2021; Ramlall, 2023). For each rule specification and scenario, I identify parameters that maximize welfare, minimize the central bank's loss, or minimize cumulative emissions, characterizing the trade-offs policymakers face.

The results reveal that the optimal standard Taylor rule takes a relatively weak stance on inflation, providing only partial accommodation to the transition's cost-push shock. Introducing an emissions response into the rule yields welfare gains of 0.02–0.03 percentage points in consumption-equivalent terms, though this benefit comes at the cost of substantially higher inflation volatility. From a stabilization perspective, however, the optimal strategy shifts; minimizing central bank loss calls for a small but positive emissions coefficient, suggesting that emissions data can serve as a valuable supply-side indicator without destabilizing prices. This policy ranking remains consistent across both surprise and anticipated-gradual carbon tax scenarios, with anticipation magnifying

both the welfare benefits and the inflationary costs of such accommodation.

This paper contributes to the debate on Central Banks’ roles in climate risk management (Battiston et al., 2017; Campiglio et al., 2018), suggesting that the optimal monetary response to carbon pricing is neither to ignore the transition shock nor to actively steer it. Rather than adopting an explicit climate mandate — a step that risks overburdening monetary policy in violation of the Tinbergen principle (Chen et al., 2021) — central banks can improve transition outcomes by treating emissions dynamics as an informative signal of underlying supply conditions. This “pragmatic augmentation” yields some welfare gains while preserving price stability, consistent with recent arguments that monetary policy should “see through” the transitory cost-push component of carbon pricing rather than either accommodating or fighting it aggressively (Olovsson and Vestin, 2023; Del Negro et al., 2023; Schnabel, 2022).

At the same time, the finding that aggressive emissions targeting generates large welfare improvements at the cost of severe nominal instability emphasizes the importance of institutional design: the welfare-optimal policy is structurally incompatible with inflation-targeting mandates, reinforcing the case for a clear division of roles in which fiscal instruments drive the emissions path, and monetary policy drives the macroeconomic adjustment (McKibbin et al., 2020; Nakov and Thomas, 2023; Coenen et al., 2024).

The remainder of the paper is organized as follows. Section 2 presents the related literature. Section 3 presents the quantitative model. Section 4 presents the Computational Strategy, and Section 5 presents the main results, including optimal parameters, welfare comparisons, and impulse-response analysis. Section 6 delivers a discussion about the main results and policy implications. Section 7 concludes.

2 Related Literature

There is a growing body of macroeconomic research that employs Dynamic Stochastic General Equilibrium (DSGE) models with environmental externalities to examine the relationship between climate policies and business cycles. These models, commonly referred to as Environmental DSGE (E-DSGE) models,¹ provide a structured framework for assessing the macroeconomic implications

¹For an extensive review of E-DSGE models and business cycles, see Annicchiarico et al. (2022).

of transitioning to a low-carbon economy. This paper aligns with those that incorporate financial frictions to analyze the effects of climate policies in the presence of credit market imperfections, with a particular focus on the role of conventional monetary policy.

A strand of this literature examines how financial frictions interact with climate policy. Carattini et al. (2023) develop an E-DSGE model with banking frictions and demonstrate that even moderate carbon tax shocks can generate financial instability, though macroprudential policy can mitigate these transition risks. My contribution is complementary but distinct in three respects. First, the focus is on *monetary* rather than macroprudential instruments. Second, we systematically optimize Taylor rule parameters across multiple objectives rather than analyzing fixed baseline rules. Third, the model incorporates endogenous abatement technology, which proves central to understanding the scope for monetary accommodation: when firms can reduce emissions intensity in response to carbon pricing, part of the adjustment occurs at the intensive margin, reducing the burden on aggregate demand management.

On conventional monetary policy and climate, McKibbin et al. (2020) analyzes how different monetary frameworks respond to climate change, finding that inflation targeting exacerbates output declines under carbon taxes while nominal income targeting mitigates these effects. Their results stress the importance of coordinating climate and monetary policy, as improper monetary responses to carbon pricing can heighten macroeconomic volatility. Chen et al. (2021) introduces a climate-augmented monetary policy rule within an E-DSGE model, finding that monetary policy must account for existing climate policies. While they explore emissions-gap targeting, which can enhance welfare but risks overburdening central banks, this paper takes a different approach by systematically comparing Taylor rule specifications across different carbon tax implementation scenarios and identifying the structural conditions under which emissions targeting is beneficial.

Recent literature has characterized the green transition primarily through the lens of production networks and relative price adjustments. Del Negro et al. (2023) employ a granular multi-sector framework to demonstrate that carbon taxation generates a sizable, albeit transitory, stagflationary trade-off. Regarding the policy response, Olovsson and Vestin (2023) argue analytically that central banks should "look through" these energy price changes by targeting core inflation, while Nakov and Thomas (2023) find that under realistic (suboptimal) carbon tax paths, monetary policy remains a tool that should prioritize strict price stability. I complement this literature by introducing a critical

transmission channel absent in these frameworks: the financial sector. By modeling the feedback loop between asset prices, bank net worth, and investment, I show that the transition is not merely a supply shock but also a financial shock. Consequently, optimal policy depends not just on the level of the carbon tax, but also on its timing (anticipated vs. surprise) and on the degree to which monetary accommodation can shield the financial sector from asset stranding while incentivizing endogenous abatement.

My focus on anticipated versus unanticipated policy implementation connects the news shocks literature (Beaudry and Portier, 2006; Schmitt-Grohé and Uribe, 2012) to recent applications in climate economics. For instance, Bartocci et al. (2024) shows that the inflationary impact of carbon taxes depends critically on the timing of household expectations, finding that anticipation can amplify near-term inflationary pressures. Similarly, Coenen et al. (2024) shows that the macroeconomic costs of the transition are highly sensitive to the credibility of the policy and the monetary reaction function. I complement these studies by optimizing policy rules across distinct implementation scenarios—rather than analyzing fixed baselines—and by explicitly identifying the role of financial frictions and endogenous abatement in propagating these 'climate news' shocks."

This paper also contributes to the literature on whether central banks should adopt expanded mandates. While the traditional view asserts that central banks should focus solely on price stability (Clarida et al., 1999; Svensson, 2000), Debortoli et al. (2019) argue that loss functions should place significant weight on economic activity. This debate has gained renewed relevance as central banks face transition risks (Acharya, 2015; Krogstrup and Oman, 2019). I show that the optimal policy specification depends on how climate policy is implemented and on the economy's structural features, suggesting that mandate considerations should be context-dependent rather than universal.

3 Model

I develop a two-sector Environmental Dynamic Stochastic General Equilibrium (E-DSGE) model with financial frictions to analyze optimal monetary policy during carbon tax transitions. The model builds on elements from Diluiso et al. (2021) and Ferrari and Nispi Landi (2024), featuring a New Keynesian framework with financial intermediation, a representative household, and two production sectors: a green sector that operates sustainably and a brown sector that emits pollutants. The

economy incorporates sector-specific capital inputs and multiple bank-held assets, with financial frictions arising from an agency problem that limits banks' ability to obtain funds from households, following Gertler and Karadi (2011). Nominal price rigidities make monetary policy central to real activity dynamics and the transmission of shocks. The sectoral structure enables the analysis of systemic spillover effects from climate policy and the associated transition risks.

Additionally, I adopt recursive Epstein–Zin preferences (Epstein and Zin, 1991; Barrage, 2020), which relax the tight link between risk aversion and the elasticity of intertemporal substitution implied by standard CRRA utility. This flexibility is particularly important in the context of climate policy, where households face long-horizon uncertainty about both economic outcomes and environmental damages. The recursive structure allows agents to form forward-looking assessments of welfare risk, amplifying the welfare consequences of carbon taxation and monetary policy responses to transition shocks. A growing literature in climate economics motivates this specification, showing that Epstein–Zin preferences can generate optimal policies and welfare implications that differ sharply from those obtained under expected utility and the Ramsey framework (Ackerman et al., 2013; Hambel et al., 2021).

Then, the model features endogenous abatement technology that allows brown firms to reduce emissions in response to carbon pricing. This margin of adjustment proves important for understanding optimal monetary policy. When firms can abate, they absorb part of the carbon tax shock by investing in cleaner production, dampening the emissions response, and reducing the burden on monetary policy. Additionally, financial frictions in the banking sector can amplify macroeconomic fluctuations during the transition.

3.1 Households

There is a continuum of infinitely lived households with preferences over consumption and leisure. Each household consists of workers and bankers. Workers supply labor and return their wages to the household, while bankers manage financial intermediaries and transfer profits to the household. Households cannot lend to intermediaries owned by their own members. As in Gertler and Karadi (2011), a fraction $1 - f$ of household members are workers and the remaining fraction f are bankers, with individuals transitioning between occupations over time at an exogenous rate that keeps proportions constant.

The representative household has recursive Epstein-Zin preferences over consumption (c_t) and leisure ($1 - L_t$):

$$V_t = \left[(1 - \beta) \left((c_t - h c_{t-1})^\omega (1 - L_t)^{1-\omega} \right)^{1-\rho} + \beta \left[\mathbb{E}_t (V_{t+1}^{1-\sigma}) \right]^{\frac{1-\rho}{1-\sigma}} \right]^{\frac{1}{1-\rho}}. \quad (1)$$

where ω is the utility weight on consumption, β is the discount factor, $\sigma > 0$ is risk aversion, and $\rho = 1/\psi$ with $\psi > 0$ being the elasticity of intertemporal substitution. The labor composite is $L_t = \left(L_{b,t}^{1+\rho_l} + L_{g,t}^{1+\rho_l} \right)^{\frac{1}{1+\rho_l}}$, where ρ_l governs the elasticity of substitution between sectors.

The household budget constraint is:

$$c_t + D_{t+1} = w_{b,t} L_{b,t} + w_{g,t} L_{g,t} + T_t + R_t D_t + \Pi_{g,t} + \Pi_{b,t} + \Phi_t \quad (2)$$

where $w_{k,t}$ is the real wage in sector k , D_t are deposits earning gross return R_t , T_t are lump-sum taxes/transfers, $\Pi_{k,t}$ are profits from sector k firms, and Φ_t are transfers from exiting bankers that serve as start-up funds for new bankers.

The first-order conditions yield labor supply equations for each sector:

$$(1 - L_t) \left(L_{b,t}^{1+\rho_l} + L_{g,t}^{1+\rho_l} \right)^{\frac{\rho_l}{1+\rho_l}} L_{k,t}^{-\rho_l} = \frac{1 - \omega}{\omega} \frac{c_t - h c_{t-1}}{w_{k,t}}, \quad k \in \{g, b\}. \quad (3)$$

The stochastic discount factor is:

$$M_{t,t+1} = \beta \frac{c_t - h c_{t-1}}{c_{t+1} - h c_t} \frac{((c_{t+1} - h c_t)^\omega (1 - L_{t+1})^{1-\omega})^{1-\rho}}{((c_t - h c_{t-1})^\omega (1 - L_t)^{1-\omega})^{1-\rho}} \left(\frac{V_{t+1}^{1-\sigma}}{\mathbb{E}_t (V_{t+1}^{1-\sigma})} \right)^{\frac{\rho-\sigma}{1-\sigma}}. \quad (4)$$

and the Euler equation is $\mathbb{E}_t [M_{t,t+1} R_{t+1}] = 1$.

3.2 Final Goods Producers

A representative final goods firm aggregates intermediate goods using a CES technology:

$$Y_t = \left[\int_0^1 y_t(i)^{\frac{\epsilon-1}{\epsilon}} di \right]^{\frac{\epsilon}{\epsilon-1}} \quad (5)$$

where $y_t(i)$ is the intermediate good produced by firm i and ϵ is the elasticity of substitution.

Profit maximization yields the demand function:

$$y_t(i) = Y_t \left(\frac{p_t(i)}{P_t} \right)^{-\epsilon} \quad (6)$$

where $p_t(i)$ is the price set by firm i and P_t is the aggregate price level.

3.3 Intermediate Producers

A continuum of intermediate firms produce differentiated goods using a CES bundle of green ($y_{g,t}$) and brown ($y_{b,t}$) inputs:

$$y_t(i) = \left[(1 - \zeta)^{\frac{1}{\gamma}} y_{g,t}(i)^{\frac{\gamma-1}{\gamma}} + \zeta^{\frac{1}{\gamma}} y_{b,t}(i)^{\frac{\gamma-1}{\gamma}} \right]^{\frac{\gamma}{\gamma-1}} \quad (7)$$

where ζ is the share of brown goods in production and γ is the elasticity of substitution between green and brown inputs. Cost minimization yields demand functions:

$$y_{g,t} = (1 - \zeta) \left(\frac{p_{g,t}}{MC_t} \right)^{-\gamma} y_t, \quad y_{b,t} = \zeta \left(\frac{p_{b,t}}{MC_t} \right)^{-\gamma} y_t \quad (8)$$

where $MC_t = \left[(1 - \zeta)p_{g,t}^{1-\gamma} + \zeta p_{b,t}^{1-\gamma} \right]^{\frac{1}{1-\gamma}}$ is the marginal cost.

In this monopolistic competition setting, firms face quadratic price adjustment costs à la Rotemberg (1982). From a symmetric equilibrium, I obtain the New Keynesian Phillips Curve:

$$\pi_t(\pi_t - \bar{\pi}) = \mathbb{E}_t \left[M_{t,t+1} \pi_{t+1} (\pi_{t+1} - \bar{\pi}) \frac{Y_{t+1}}{Y_t} \right] + \frac{\epsilon}{\kappa_P} \left(MC_t - \frac{\epsilon - 1}{\epsilon} \right) \quad (9)$$

where π_t is inflation, $\bar{\pi}$ is steady-state inflation, and κ_P is the price adjustment cost parameter.

3.4 Green and Brown Producers

Both sectors produce using Cobb-Douglas technology with capital and labor:

$$y_{k,t} = A_t (\xi_{k,t} K_{k,t-1})^\alpha L_{k,t}^{1-\alpha}, \quad k \in \{g, b\} \quad (10)$$

where A_t is aggregate productivity affected by pollution through a damage function standard

in DICE models (Nordhaus, 2017): $A_t = (1 - (D_0 + D_1 X_t + D_2 X_t^2))a_t$, with a_t following an AR(1) process. The term $\xi_{k,t}$ is a sector-specific capital quality shock.

Following Heutel (2012), only the brown sector generates emissions. The stock of pollution evolves according to:

$$X_t = (1 - \delta_x)X_{t-1} + e_t + e^* \quad (11)$$

where e_t are emissions from brown production, δ_x is the natural decay rate, and e^* represents emissions from the rest of the world.

Emissions depend on brown output and the level of abatement:

$$e_t = (1 - ab_t)y_{b,t}^{1-\lambda} \quad (12)$$

where $ab_t \in [0, 1]$ is the abatement rate and λ governs the elasticity of emissions with respect to output.

Brown firms maximize profits by choosing labor, capital, and abatement. They finance capital purchases by issuing bonds $b_{b,t} = q_{b,t}K_{b,t}$ and face a carbon tax τ_t on emissions. Abatement is costly:

$$Z_t = \theta_1 ab_t^{\theta_2} y_{b,t} \quad (13)$$

where θ_1 and θ_2 govern abatement cost efficiency.

The brown firm's problem is:

$$\max \quad p_{b,t}y_{b,t} - w_{b,t}L_{b,t} - r_{kb,t}K_{b,t-1} - \tau_t e_t - Z_t$$

where $r_{kb,t} = r_{b,t}q_{b,t-1} - (1 - \delta)q_{b,t}$ is the rental rate of capital. The first-order conditions determine labor demand, capital demand, and optimal abatement:

$$(1 - \alpha)y_{b,t} \left(p_{b,t} - \tau_t(1 - ab_t)(1 - \lambda)y_{b,t}^{-\lambda} - \theta_1 ab_t^{\theta_2} \right) = w_{b,t}L_{b,t} \quad (14)$$

$$\alpha y_{b,t} \left(p_{b,t} - \tau_t(1 - ab_t)(1 - \lambda)y_{b,t}^{-\lambda} - \theta_1 ab_t^{\theta_2} \right) = r_{kb,t}K_{b,t-1} \quad (15)$$

The optimal abatement condition equates the marginal cost of abatement to the marginal benefit

from avoided carbon taxes:

$$\theta_1 \theta_2 a b_t^{\theta_2 - 1} = \tau_t y_{b,t}^{-\lambda} \quad (16)$$

Green firms face the same production structure but are not subject to carbon taxes or abatement costs:

$$(1 - \alpha) y_{g,t} p_{g,t} = w_{g,t} L_{g,t}, \quad \alpha y_{g,t} p_{g,t} = r_{kg,t} K_{g,t-1} \quad (17)$$

3.5 Capital Producers

Capital producers purchase final goods and undepreciated capital to produce new capital for both sectors. They face investment adjustment costs and solve:

$$\max \mathbb{E}_0 \sum_{t=0}^{\infty} M_{0,t} \sum_{k \in \{g,b\}} [q_{k,t} K_{k,t} - (1 - \delta) q_{k,t} K_{k,t-1} - I_{k,t}]$$

$$\text{s.t. } K_{k,t} = (1 - \delta) K_{k,t-1} + \left[1 - \frac{\kappa_I}{2} \left(\frac{I_{k,t}}{I_{k,t-1}} - 1 \right)^2 \right] I_{k,t}$$

where $q_{k,t}$ is the price of capital in sector k , δ is depreciation, and κ_I governs adjustment costs.

The first-order condition is:

$$1 = q_{k,t} \left(1 - \frac{\kappa_I}{2} \left(\frac{I_{k,t}}{I_{k,t-1}} - 1 \right)^2 - \kappa_I \left(\frac{I_{k,t}}{I_{k,t-1}} - 1 \right) \frac{I_{k,t}}{I_{k,t-1}} \right) + \mathbb{E}_t \left[M_{t,t+1} \kappa_I q_{k,t+1} \left(\frac{I_{k,t+1}}{I_{k,t}} - 1 \right) \left(\frac{I_{k,t+1}}{I_{k,t}} \right)^2 \right] \quad (18)$$

3.6 Financial Intermediaries

Financial intermediaries introduce financial frictions that can amplify macroeconomic fluctuations during the carbon transition. I follow Gertler and Karadi (2011), with the adaptations from Ferrari and Nispi Landi (2024) and Carattini et al. (2023) in modeling this sector.

Including financial frictions is crucial for understanding transition risk. When carbon-intensive sectors face policy shocks, the value of brown assets declines, reducing bank collateral. This deterioration in balance sheets leads to tighter credit conditions, higher financing costs, and potentially a contraction in economic activity. Carattini et al. (2023) show that an economy with financial frictions experiences a deeper recession than a frictionless economy, amplifying transition risks.

Banks raise deposits from households and lend to green and brown producers. The balance sheet of the representative bank is:

$$b_{g,t}^f + b_{b,t}^f = NW_t + D_t \quad (19)$$

where $b_{k,t}^f$ are bonds held on firms in sector k , NW_t is bank net worth, and D_t are deposits.

To generate a spread between green and brown bond rates, I assume banks face a quadratic cost when deviating from a target portfolio share b^* :

$$NW_t = r_{g,t} b_{g,t-1}^f + r_{b,t} b_{b,t-1}^f - \frac{r_{t-1}}{\pi_t} D_{t-1} - \frac{\kappa_F}{2} NW_{t-1} \left[\frac{b_{g,t-1}^f}{b_{t-1}^f} - b^* \right]^2 \quad (20)$$

This cost captures real-world frictions in portfolio reallocation, such as regulatory compliance costs or internal risk management policies. It creates a wedge between green and brown returns, preventing instantaneous adjustment to market conditions.

An agency problem limits bank leverage. Bankers can divert a fraction θ of assets, so depositors require that the bank's continuation value exceeds the diversion benefit:

$$V_t^f \geq \theta (b_{g,t}^f + b_{b,t}^f) \quad (21)$$

where the bank's value is:

$$V_t^f = \mathbb{E}_t \left[\sum_{i=0}^{\infty} (1 - \chi) \chi^i M_{t,t+i} NW_{t+1+i} \right] \quad (22)$$

and χ is the probability a banker continues operating.

Assuming the constraint binds, the first-order conditions determine bank leverage $lev_t = b_t^f / NW_t$ and green leverage $lev_t^g = b_{g,t}^f / NW_t$:

$$lev_t = \frac{\mathbb{E}_t \left[M_{t,t+1} \nu_{t+1} \left(\frac{r_t}{\pi_{t+1}} + (r_{g,t+1} - r_{b,t+1}) lev_{g,t} - \frac{\kappa_F}{2} \left(\frac{lev_{g,t}}{lev_t} - b^* \right)^2 \right) \right]}{\theta - \mathbb{E}_t \left[M_{t,t+1} \nu_{t+1} \left(r_{b,t+1} - \frac{r_t}{\pi_{t+1}} \right) \right]} \quad (23)$$

$$\mathbb{E}_t \left[M_{t,t+1} \nu_{t+1} \left(r_{g,t+1} - r_{b,t+1} - \left(\frac{lev_{g,t}}{lev_t} - b^* \right) \frac{\kappa_F}{lev_t} \right) \right] = 0 \quad (24)$$

where ν_t is the bank's marginal value of net worth.

Aggregate bank net worth combines continuing and new bankers:

$$NW_t = \chi \left(\frac{r_{t-1}}{\pi_t} + \left(r_{b,t} - \frac{r_{t-1}}{\pi_t} \right) lev_{t-1} + (r_{g,t} - r_{b,t}) lev_{t-1}^g - \frac{\kappa_F}{2} \left(\frac{lev_{g,t-1}}{lev_{t-1}} - b^* \right)^2 \right) NW_{t-1} + \iota b_t^f \quad (25)$$

where ι is the fraction of assets transferred to new bankers.

3.7 Monetary Policy

Monetary policy is conducted through Taylor-type interest rate rules with interest rate smoothing.

In the baseline specification, the central bank follows a standard rule,

$$\frac{r_t}{\bar{r}} = \left(\frac{r_{t-1}}{\bar{r}} \right)^{\rho_r} \left[\left(\frac{\pi_t}{\bar{\pi}} \right)^{\phi_\pi} \left(\frac{Y_t}{\bar{Y}} \right)^{\phi_y} \right]^{1-\rho_r}, \quad (26)$$

where r_t denotes the nominal policy rate, \bar{r} its steady-state value, π_t inflation, $\bar{\pi}$ the inflation target, and ρ_r captures interest rate smoothing. The coefficients ϕ_π and ϕ_y govern the policy response to inflation and output fluctuations induced by the carbon tax.

To study monetary policy responses to transition risk, I augment the rule to allow for systematic reactions to emissions dynamics:

$$\frac{r_t}{\bar{r}} = \left(\frac{r_{t-1}}{\bar{r}} \right)^{\rho_r} \left[\left(\frac{\pi_t}{\bar{\pi}} \right)^{\phi_\pi} \left(\frac{Y_t}{\bar{Y}} \right)^{\phi_y} \left(\frac{\sum_{j=1}^4 e_{t-j}}{\sum_{j=5}^8 e_{t-j}} \right)^{\phi_e} \right]^{1-\rho_r}. \quad (27)$$

The coefficient ϕ_e measures the sensitivity of the policy rate to emissions momentum. A positive value implies policy accommodation when emissions decline, a salient feature of a low-carbon transition. Through this channel, monetary policy supports asset prices and mitigates balance-sheet stress as carbon-intensive activity contracts.

The rule targets the rate of change in emissions rather than deviations from a fixed steady state. During the transition, the economy converges toward a new, lower-emissions equilibrium, making the initial steady state an inappropriate benchmark. Focusing on emissions growth allows the central bank to manage the pace of adjustment without resisting the underlying structural shift. The lag structure reflects the delayed availability of environmental data in practice.

Emissions are not treated as an independent policy objective, nor as a substitute for climate policy. Instead, they serve as a transition-specific indicator of supply-side pressure that is imperfectly captured by inflation or the output gap. Carbon pricing alters relative production costs across sectors, and these reallocations appear in emissions data before they are fully reflected in aggregate prices. A positive response to declining emissions therefore represents state-contingent accommodation of transition-related supply disturbances rather than an attempt to directly steer climate outcomes.

Finally, while Ramsey policy provides a theoretical welfare benchmark, it relies on full commitment to a state-contingent policy path, which is implausible in the presence of policy uncertainty and implementation lags associated with climate policy announcements. By optimizing simple and implementable Taylor rules, the analysis identifies robust monetary responses that perform well even when expectations cannot be perfectly managed during the pre-announcement phase of the transition.

3.8 Government and Market Clearing

The government implements the carbon tax τ_t and rebates revenues lump-sum to households through T_t^2 . The carbon tax follows one of three transition paths corresponding to my scenarios: an unanticipated permanent increase, an anticipated increase announced four quarters in advance, or a gradual phase-in over twenty quarters.

The final goods market clears:

$$Y_t = c_t + I_t + \frac{\kappa_F}{2} NW_{t-1} \left[\frac{b_{g,t-1}^f}{b_{t-1}^f} - b^* \right]^2 + \frac{\kappa_P}{2} (\pi_t - \bar{\pi})^2 Y_t + Z_t \quad (28)$$

where $I_t = I_{g,t} + I_{b,t}$ is aggregate investment and Z_t is abatement costs.

Bond markets clear with $b_{k,t} = b_{k,t}^f$ for $k \in \{g, b\}$, and labor markets clear in each sector.

²The assumption of lump-sum rebates provides a clean welfare benchmark by abstracting from distortionary tax interactions. Alternative recycling schemes—such as labor tax cuts or green investment subsidies—would alter both the fiscal multiplier of the carbon tax and potentially the optimal monetary response. Formal analysis of optimal recycling under different monetary regimes remains an important avenue for future research.

3.9 Carbon Tax Policy and Transition Scenarios

The primary driver of the low-carbon transition in the model is an exogenous carbon tax, τ_t , levied on the emissions generated by the brown sector. This tax modifies the marginal cost structure of brown firms, incentivizing decarbonization through two distinct channels: an extensive margin (reducing total brown output) and an intensive margin (investing in abatement technology to lower emissions per unit of output). To isolate the allocative efficiency of the policy from wealth distribution effects, all revenues generated by the carbon tax are rebated lump-sum to households. The evolution of the carbon tax is governed by an AR(1) process that allow two different scenarios, a surprise shock and a staggered implementation schedule, where anticipated policy changes are phased in gradually over time rather than occurring as a single discrete jump.

The tax process is given by:

$$\tau_t = (1 - \rho_\tau)\bar{\tau} + \rho_\tau\tau_{t-1} + \varepsilon_t^{\tau,surprise} + \sum_{k=4}^8 \psi_k \varepsilon_{t-k}^{\tau,Gradual} \quad (29)$$

where $\bar{\tau}$ represents the steady-state carbon tax rate and ρ_τ governs the persistence of the tax. The term ε_t^τ represents a standard unanticipated shock. The news component, $\varepsilon_t^{\tau,news}$, represents a policy announced at time t but implemented according to a phase-in schedule defined by the weights ψ_k . I calibrate the weights to $\psi_k = 0.25$ for $k \in \{4, \dots, 8\}$, satisfying $\sum \psi_k = 1$. This implies that the full magnitude of the announced policy is gradually incorporated into the tax rate between quarters 4 and 8.

This specification allows me to analyze two distinct transition scenarios:

1. **Surprise Transition** ($\varepsilon_t^{\tau,surprise}$): An immediate, unanticipated hike in the carbon tax. This scenario mimics a sudden shift in climate policy (e.g., an executive order), forcing firms to adjust production and prices instantly without prior capital reallocation.
2. **Anticipated Phase-In** ($\varepsilon_{t-k}^{\tau,Gradual}$): A credible policy announcement where the carbon tax is legislated today but implemented progressively starting one year (4 quarters) later. The tax burden increases over the subsequent year (quarters 4 through 8). This formulation captures the realistic "implementation lags" of climate legislation, creating a prolonged window for intertemporal substitution where agents can adjust investment and consumption in anticipation

of the rising cost path.

This staggered structure creates a "news trap" where inflationary pressures from forward-looking pricing ($t = 0$) precede the actual supply contraction ($t \geq 4$), testing the central bank's ability to "look through" the anticipation phase while managing the gradual tightening of supply conditions.

The shock size is chosen so that the carbon tax approximately doubles from its initial steady-state value, consistent with the magnitude of carbon price increases contemplated in major climate policy proposals. The final carbon tax represents (approximately \$35 per ton of CO_2), consistent with the global average carbon price in 2024 and within the range of carbon taxes implemented in Europe. The conversion follows Carattini et al. (2023), who map DSGE model units to dollar values using U.S. GDP and emissions data ³.

The horizon for welfare and emissions evaluation is set to 40 quarters, allowing sufficient time for the economy to adjust to the carbon tax level while remaining within a policy-relevant time frame.

3.10 Assessment Criteria

The central question of this paper is how the optimal values of ϕ_π , ϕ_y , and ϕ_e depend on: (i) how the carbon tax is implemented (unanticipated, anticipated); (ii) the availability of abatement technology; and (iii) the presence of financial frictions.

To evaluate monetary policy performance, I use different criteria. First, household welfare is measured by the recursive specification of Epstein-Zin preferences V_t , where I compute the change from the Stochastic Steady state in Consumption Equivalent Units.

Second, a central bank loss function following Debortoli et al. (2019):

$$L_t^{CB} = \lambda_\pi(\pi_t - \bar{\pi})^2 + \lambda_{\Delta r}(r_t - r_{t-1})^2 + \lambda_y(Y_t - \bar{Y})^2 \quad (30)$$

where the weights $\lambda_\pi = 1$, $\lambda_{\Delta r} = 0.077$, and $\lambda_y = 1.042$ reflect priorities consistent with dual-mandate central banks. The inclusion of interest rate volatility ensures gradual policy adjustments,

³While leading carbon pricing jurisdictions have exceeded €80/ton (EU ETS in 2023), our calibration targets the global average carbon price. The robustness analysis in Table 6 demonstrates that for larger shocks, the optimal emissions response converges toward zero as cost-push inflation dominates, suggesting aggressive accommodation is feasible only for moderate carbon price increases.

while the weight on output acknowledges trade-offs in responding to supply shocks.

Comparing results under the welfare criterion and the central bank loss function reveals whether there are trade-offs between household welfare and traditional macroeconomic stabilization objectives. Such trade-offs are policy-relevant, as they indicate whether a central bank focused narrowly on its traditional mandate would choose different policies than one seeking to maximize household welfare.

Finally, as the goal of the transition is to move to a low-carbon economy, I assess the discounted sum of emissions over 10 years. With this complementary measure, one can assess whether there are trade-offs between welfare, macroeconomic stability, and emissions reduction

3.11 Calibration

The model is calibrated at a quarterly frequency for the U.S. economy. I draw on the extensive DSGE estimation literature, particularly Smets and Wouters (2007) and Gertler and Karadi (2011), supplemented with parameters from the environmental macroeconomics literature (Heutel, 2012; Carattini et al., 2023; Nordhaus, 2017).

Tables 1 and 2 summarize the complete calibration.

Table 1: Macroeconomic Parameters

Parameter	Description	Value	Source
<i>Preferences</i>			
β	Discount factor	0.995	Smets and Wouters (2007)
h	Consumption Habit	0.70	Smets and Wouters (2007)
σ	Risk aversion	10	Pohl et al. (2018)
ψ	Intertemporal elasticity of substitution	1.5	Pohl et al. (2018)
ω	Consumption utility weight	0.32	Barrage (2020)
<i>Production</i>			
α	Capital share	0.33	Smets and Wouters (2007)
δ	Capital depreciation rate	0.025	Smets and Wouters (2007)
ε	Elasticity of substitution	6	Christiano et al. (2005)
κ_p	Rotemberg price stickiness	58.53	Match calvo = 0.75
κ_I	Investment adjustment cost	5.5	Smets and Wouters (2007)
<i>Monetary Policy</i>			
$\bar{\pi}$	Target inflation rate (quarterly)	1.005	2% Federal Reserve Target
ρ_r	Interest rate smoothing	0.80	Smets and Wouters (2007)
<i>Financial Frictions</i>			
χ	Banker survival probability	0.965	Gertler and Karadi (2011)
θ	Divertible asset share	Calibrated	Match $\phi_{ss} = 6$
ι	New banker wealth share (=0.003)	Calibrated	Match spread = 200 bps

Risk aversion is set to $\sigma = 10$ and $\psi = 1.5$ following the long-run risks literature (Pohl et al., 2018) and climate-economy models under uncertainty (Cai and Lontzek, 2019), implying precaution-

ary savings and significant risk premia. The intertemporal elasticity of substitution ψ is set to 1.5 to capture empirically plausible consumption dynamics. This ensure agents prefer early resolution of uncertainty.

The elasticity of substitution across varieties $\varepsilon = 6$ implies a steady-state markup of 20%, following Christiano et al. (2005). Price stickiness is governed by a Calvo parameter $\theta_p = 0.75$, implying an average price duration of four quarters, consistent with Nakamura and Steinsson (2008).

The baseline Taylor rule parameters follow standard estimates: an interest rate smoothing $\rho_r = 0.80$ (Smets and Wouters, 2007), and the inflation target is the Federal Reserve’s 2% annual rate.

Following Gertler and Karadi (2011), banker survival probability is $\chi = 0.965$, implying an average banker tenure of eight years. The divertibility parameter θ is calibrated to match a steady-state leverage ratio of 6, reflecting post-Dodd-Frank regulatory constraints. The wealth share for new bankers ι targets a corporate credit spread of 200 basis points annually.

The climate damage function follows the DICE model (Nordhaus, 2017):

$$D(X_t) = D_0 + \tilde{D}_1 X_t + \tilde{D}_2 X_t^2 \tag{31}$$

where X_t is the atmospheric CO₂ concentration in GtC. Following Ferrari and Nispi Landi (2024), I set $D_0 = -0.0076$, $\tilde{D}_1 = 8.10 \times 10^{-6}$, and $\tilde{D}_2 = 1.05 \times 10^{-8}$, which implies approximately 2.7% output loss at 3°C warming.

Table 2: Environmental and Sectoral Parameters

Parameter	Description	Value	Source
<i>Climate Damages</i>			
D_0	Damage constant	-0.0076	Nordhaus (2017)
\tilde{D}_1	Damage linear term	8.10×10^{-6}	Nordhaus (2017)
\tilde{D}_2	Damage quadratic term	1.05×10^{-8}	Nordhaus (2017)
δ_x	Pollution decay rate	0.0035	Nordhaus (2017)
<i>Green-Brown Structure</i>			
γ	Green-brown substitution	2	Carattini et al. (2023)
ζ	Brown goods weight	0.8	Giovanardi et al. (2023)
κ_G	Green capital share	0.20	U.S. renewable capacity
ψ	Emissions-output elasticity	0.4	Heutel (2012)
e^{row}	Rest-of-world emissions ratio	8	Global emissions data
<i>Abatement</i>			
θ_1	Abatement cost scale	0.0334	Nordhaus (2017)
θ_2	Abatement cost curvature	2.6	Nordhaus (2017)

Green and brown goods exhibit imperfect substitutability with elasticity $\gamma = 2$ (Carattini et al.,

2023). The initial green capital share is $\kappa_G = 0.20$, reflecting current renewable energy capacity in U.S. electricity generation. Emissions intensity follows $\psi = 0.4$ (Ferrari and Nispi Landi, 2024). The pollution decay rate $\delta_x = 0.0035$ quarterly corresponds to an atmospheric CO₂ half-life of approximately 50 years. The rest-of-world emissions ratio is 8, reflecting that U.S. emissions (~ 5 GtCO₂/year) represent roughly 13.5% of global emissions (~ 37 GtCO₂/year).

3.12 Model Validation

I assess the model’s ability to reproduce key unconditional business-cycle moments observed in the data. This serves as a consistency check to ensure that the model generates plausible macroeconomic dynamics along standard dimensions of volatility, persistence, and comovement.

Table 3 reports business-cycle moments from U.S. data alongside the moments generated by the model. To facilitate comparison, relative volatilities are normalized by the output standard deviation, and persistence is measured by first-order autocorrelations. The model moments correspond to averages across 10,000 stochastic simulations under the baseline calibration, with a traditional Taylor rule calibrated to the US economy, as in Smets and Wouters (2007): $\phi_\pi = 2$, $\phi_y = 0.08$, and $\rho_r = 0.8$

The model replicates the hierarchy of volatilities observed in the data. As shown in Panel A, the model’s standard deviation of output (0.0158) is very close to the empirical counterpart (0.0149). Investment is substantially more volatile than output (relative standard deviation of 3.88), closely tracking the empirical ratio of 4.21. Notably, the model matches the relative volatility of hours worked exactly (1.37 in both the model and the data), suggesting that the specification of labor supply and nominal rigidities appropriately captures labor market fluctuations. Consumption in the model is somewhat smoother than in the data (relative volatility of 0.75 versus 0.91), a common feature in medium-scale DSGE models with habit formation and financial frictions (Smets and Wouters, 2007; Christiano et al., 2014).

Regarding comovement and persistence (Panel B), the model reproduces the strong persistence of real variables, with the autocorrelation of output (0.89) and investment (0.93) aligning well with empirical values. Investment and consumption are strongly procyclical, matching the data. Hours worked are also procyclical (correlation of 0.65), though slightly less strongly correlated with output than in the data (0.86).

Table 3: Business Cycle Moments: Empirical Data vs. Model

Panel A: Standard Deviations and Relative Volatilities

Variable	Standard Deviation (σ)			Relative SD (σ_x/σ_y)		
	Data	Model	[5%, 95%]	Data	Model	[5%, 95%]
Output (Y)	0.0149	0.0158	[0.0131, 0.0189]	1.00	1.00	—
Consumption (C)	0.0135	0.0118	[0.0099, 0.0137]	0.91	0.75	[0.64, 0.86]
Investment (I)	0.0626	0.0616	[0.0480, 0.0773]	4.21	3.88	[3.46, 4.36]
Hours (H)	0.0203	0.0215	[0.0185, 0.0247]	1.37	1.37	[1.16, 1.60]
Inflation (π)	0.0058	0.0134	[0.0116, 0.0154]	0.39	0.85	[0.69, 1.04]
Interest Rate (R)	0.0091	0.0062	[0.0048, 0.0077]	0.61	0.39	[0.29, 0.52]

Panel B: Persistence and Cyclicity

Variable	Autocorrelation AR(1)			Correlation with Output $\text{Corr}(x, Y)$		
	Data	Model	[5%, 95%]	Data	Model	[5%, 95%]
Output (Y)	0.77	0.89	[0.86, 0.92]	1.00	1.00	—
Consumption (C)	0.73	0.85	[0.81, 0.88]	0.88	0.74	[0.63, 0.83]
Investment (I)	0.81	0.93	[0.91, 0.95]	0.87	0.91	[0.88, 0.94]
Hours (H)	0.81	0.76	[0.70, 0.82]	0.86	0.65	[0.52, 0.75]
Inflation (π)	0.87	0.80	[0.73, 0.85]	0.15	0.43	[0.28, 0.57]
Interest Rate (R)	0.97	0.96	[0.94, 0.98]	0.17	0.21	[0.04, 0.39]

Note: This table reports the theoretical moments from the model simulation compared to the empirical moments. The "Model" column reports the mean statistic across simulations. The brackets [5%, 95%] represent the 90% confidence intervals generated by the model.

On the nominal side, inflation dynamics exhibit higher volatility and procyclicality than observed in the data. This excess volatility is a standard property of New Keynesian models that are not estimated to target unconditional nominal moments; however, the persistence of inflation (0.80) is consistent with empirical evidence (0.87).

The model captures the cyclical behavior of the short-term interest rate remarkably well. The correlation between the interest rate and output in the model is 0.21, which is close to the empirical value of 0.17. This indicates that the monetary policy rule in the model correctly captures the systematic response of the policy rate to the business cycle, validating the baseline policy specification for the subsequent analysis of transition risks.

4 Computational Strategy and Optimal Policy Search

To identify the optimal monetary policy response to the carbon transition, I employ a two-stage computational strategy that bridges standard linear-quadratic stabilization methods with non-linear welfare analysis. This approach allows me to robustly characterize the trade-offs between macroeco-

conomic stability, emissions reduction, and household welfare under different Taylor rule specifications.

4.1 Step 1: Optimal Simple Rules (OSR)

In the first stage, I establish a baseline by computing the Optimal Simple Rule (Schmitt-Grohé and Uribe, 2007). This method relies on a first-order approximation of the model dynamics and minimizes a weighted sum of unconditional variances. I consider the model loss functions to characterize the central bank’s objective derived in Debortoli et al. (2019):

$$L_{CB} = \text{Var}(\pi_t) + 1.042 \cdot \text{Var}(y_{gap,t}) + 0.077 \cdot \text{Var}(dr_t)$$

I optimize the coefficients of the standard Taylor rule (ϕ_π, ϕ_y) and the emissions-augmented rule $(\phi_\pi, \phi_y, \phi_e)$. This linear optimization provides a convex initialization point for the subsequent non-linear analysis.

4.2 Step 2: Generalized Impulse Response Optimization

Linear approximations may fail to capture the full welfare implications of the transition, particularly given the presence of Epstein-Zin preferences and the non-linearities introduced by financial frictions and abatement technology. Therefore, in the second stage, I solve the model using a second-order perturbation with pruning, which is a suitable approach for analyzing welfare in a dynamic setting (Schmitt-Grohé and Uribe, 2004). I then perform a grid search over the policy parameter space, centered on the OSR results, to identify the rule that optimizes performance conditional on the specific carbon tax shock. For each parameter combination in the grid $(\phi_\pi, \phi_y, \phi_e)$, I compute Generalized Impulse Response Functions (GIRFs) by simulating the economy with and without the carbon tax shock over 2000 Monte Carlo replications. This effectively marginalizes out the influence of concurrent stochastic volatility. I evaluate the optimal policy according the distinct criteria:

This dual approach provides that the identified optimal rules are robust to both general business cycle fluctuations (via OSR) and the specific structural break induced by the carbon transition (via GIRF).

5 Results: Optimal Monetary Policy under Transition Shocks

This section presents the main findings from the optimal monetary policy analysis during a carbon tax transition. We compare a standard Taylor rule, in which policy responds only to inflation and the output gap, with an emissions-augmented rule that additionally responds to emissions dynamics. The analysis evaluates both rules under two objectives: maximizing household welfare, measured in consumption-equivalent terms, and minimizing a conventional quadratic central bank loss function. Table 4 reports the optimal policy coefficients and associated outcomes, while Table 5 summarizes the relative gains and losses from augmenting the policy rule.

Table 4: Optimal Taylor Rule Parameters and Outcomes: Standard ($\phi_e = 0$) vs Emissions-Augmented (ϕ_e free)

Method	Criterion	ϕ_π	ϕ_y	ϕ_e	CE (%)	$L_{CB} \times 100$	e_{cum}
<i>Panel A: OSR (Order=1, Unconditional Variance)</i>							
OSR (L_{CB})	Standard	1.14	0.11	—	—	Var=0.26	—
OSR (L_{CB})	Augmented	1.15	0.11	0.014	—	Var=0.23	—
<i>Panel B: GIRF (Order=2, Surprise Shock $\varepsilon_\tau^{surprise}$)</i>							
GIRF	Std (Max Welf)	1.10	0.15	0.000	-0.099	0.047	-4.270
GIRF	Std (Min L_{CB})	1.10	0.10	0.000	-0.100	0.044	-4.275
GIRF	Aug (Max Welf)	1.50	0.12	0.100	-0.075	0.303	-4.200
GIRF	Aug (Min L_{CB})	1.50	0.25	0.025	-0.096	0.042	-4.266
<i>Panel C: GIRF (Order=2, Gradual Shock $\varepsilon_\tau^{gradual}$)</i>							
GIRF	Std (Max Welf)	1.10	0.15	0.000	-0.100	0.049	-3.687
GIRF	Std (Min L_{CB})	1.10	0.10	0.000	-0.100	0.045	-3.693
GIRF	Aug (Max Welf)	1.10	0.30	0.075	-0.073	0.187	-3.625
GIRF	Aug (Min L_{CB})	1.40	0.15	0.025	-0.097	0.043	-3.684

Notes: Standard Taylor rules restrict $\phi_e = 0$. Augmented rules allow ϕ_e to vary. Columns report: (1) Welfare as Consumption Equivalent (CE %), (2) Cumulative Discounted CB Loss (L_{CB}), and (3) Cumulative Discounted Emissions (e_{cum}). GIRF metrics computed under order=2 pruned simulation.

5.1 Optimal Policy Coefficients

Across all specifications, the standard Taylor rule delivers a strikingly consistent pattern. When the emissions coefficient is constrained to zero, the optimal inflation response lies close to the lower bound of the admissible range, with ϕ_π between 1.10 and 1.14, and the output-gap coefficient between 0.10 and 0.15. This configuration reflects the supply-side nature of the carbon tax shock. Aggressive inflation stabilization would exacerbate the output contraction induced by higher pro-

duction costs, while a more accommodative stance allows inflation to absorb part of the adjustment. As a result, the standard rule effectively accommodates the transition, but at the cost of non-trivial welfare losses of about 0.10 percent in consumption-equivalent terms.

Allowing the emissions coefficient to vary alters the optimal policy design in a systematic way. In all cases, the optimized emissions response is positive, ranging from a small value under the first-order OSR solution to substantially larger values under second-order GIRF optimization when welfare is the objective. A positive emissions coefficient implies that policy eases as emissions fall during the transition, providing additional demand support when carbon pricing tightens firms' cost conditions. At the same time, the augmented rules are paired with higher inflation coefficients, often in the range of 1.40 to 1.50. This combination indicates that emissions accommodation and inflation vigilance act as complements rather than substitutes: explicit recognition of the transition shock allows monetary policy to lean more aggressively against inflation without inducing excessive real contraction.

5.2 Welfare and Stabilization Trade-offs

Table 5 reports the trade-offs associated with augmenting the policy rule with emissions. When welfare is the objective, the augmented rule delivers positive but modest gains. Relative to the standard welfare-optimal rule, consumption-equivalent welfare increases by 0.024 percentage points following a surprise carbon tax shock and by 0.026 percentage points under an anticipated (gradual) shock. The larger gain in the latter case reflects the interaction between forward-looking adjustment and monetary accommodation ⁴.

Table 5: Gains/Losses of Emissions-Augmented Rule Relative to Standard Rule Benchmarks ($\phi_e = 0$)

Comparison	Surprise Shock			Anticipated Shock		
	ΔCE (pp)	ΔL_{CB} (%)	Δe (%)	ΔCE (pp)	ΔL_{CB} (%)	Δe (%)
Aug (Max Welfare) vs Std (Max Welfare)	0.024	-549.69	1.6	0.026	-280.46	1.7
Aug (Min L_{CB}) vs Std (Min L_{CB})	0.004	4.38	0.2	0.003	4.71	0.2

Notes: ΔCE is the difference in welfare (consumption-equivalent) in percentage points. ΔL_{CB} is the percent reduction in discounted CB loss. Δe is the percent reduction in cumulative emissions.

⁴To contextualize the magnitude of these welfare gains, Schmitt-Grohé and Uribe (2007) find that moving from sub-optimal to welfare-maximizing simple rules yields gains of 0.01–0.05 percentage points in similar DSGE frameworks. The estimates of 0.02–0.03 percentage points fall squarely within this range, suggesting the gains from emissions augmentation are economically meaningful, comparable to removing moderate macroeconomic volatility.

The mechanism behind these gains is intertemporal smoothing. A positive emissions coefficient implies that policy eases as emissions fall during the transition, partially offsetting the contractionary effects of higher production costs and supporting consumption during the adjustment. With Epstein-Zin preferences, which place a high premium on reducing short-run consumption risk, even small improvements in the consumption path translate into non-negligible welfare gains. These gains reflect a smoother transition rather than higher long-run consumption.

The welfare improvements under aggressive emissions accommodation are obtained at the expense of traditional stabilization performance. In both shock environments, the welfare-maximizing augmented rule raises the discounted central bank loss sharply relative to the standard rule, with L_{CB} worsening by roughly 280 to 550 percent. This deterioration reflects higher inflation and output-gap volatility because monetary policy is accommodating a cost-push disturbance rather than leaning against it. From the perspective of an institution primarily charged with price stability, this policy would be difficult to reconcile with a traditional mandate.

When the objective is to minimize L_{CB} , the optimal augmented rule becomes substantially more conservative. The emissions response remains positive but small, and the rule places greater weight on inflation stabilization. In this case, augmentation modestly improves welfare, with ΔCE between 0.003 and 0.004 percentage points, while reducing the central bank loss by about 4 to 5 percent. The implied environmental differences are negligible, with cumulative emissions changing by only about 0.2 percent relative to the standard loss-minimizing benchmark. Thus, a limited emissions response can yield small welfare improvements and better stabilization outcomes without altering the emissions path. In practice, this implies that the central bank should not attempt to actively drive the climate transition, but can treat emissions dynamics as an auxiliary indicator of supply-side pressures, using them to fine-tune policy without compromising price stability or environmental policy objectives.

These results do not imply that optimal monetary policy should disregard inflationary pressures associated with the green transition. On the contrary, inflation plays a central role in the transmission of the carbon tax shock across all policy regimes. The welfare-maximizing augmented rule accommodates inflation temporarily as a deliberate mechanism to smooth real adjustment costs in the presence of a supply-driven price increase, not because inflation stabilization is unimportant. This accommodation comes at the cost of large and persistent deviations of inflation from target,

which explains the sharp deterioration in the central bank loss under welfare maximization. When the policy objective instead reflects a conventional stabilization mandate, the optimal response tightens inflation control substantially, and the emissions coefficient becomes small. In this case, inflation remains well contained, nominal volatility is limited, and welfare gains from emissions augmentation are correspondingly modest. The contrast between these regimes stresses that greenflation is not neglected in the analysis, but rather explicitly traded off against real stabilization, with mandate-consistent policies placing primary weight on price stability.

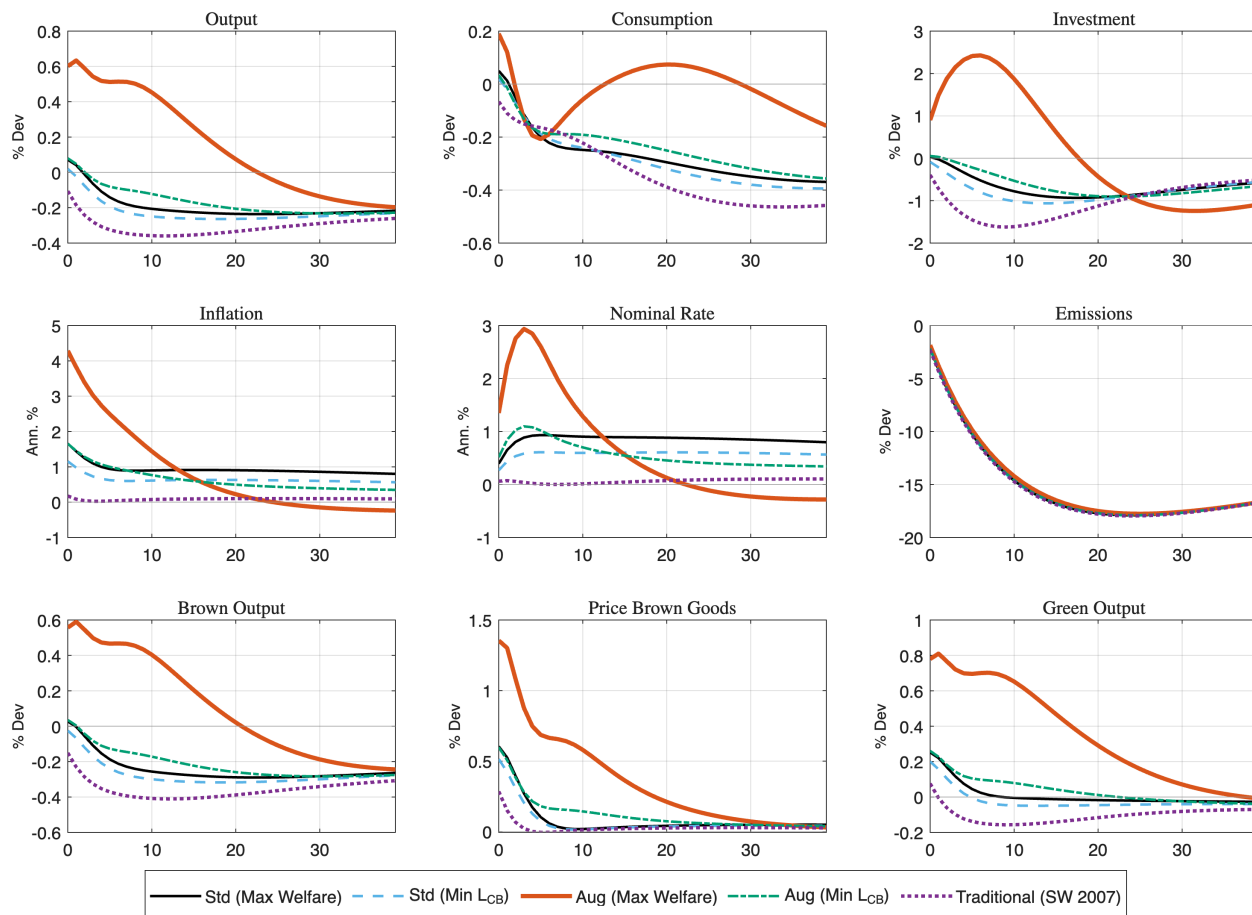
5.3 Dynamic Adjustment and Transmission

The impulse response functions in Figures 1 and 2 illustrate the distinct transmission mechanisms across the five policy regimes. The inclusion of a traditional Taylor rule benchmark ($\phi_\pi = 2.0, \phi_y = 0.08$) (Smets and Wouters, 2007) emphasizes the role of monetary strictness in shaping the transition. Under this strict inflation-targeting regime, the carbon tax acts as a pure supply shock: the central bank raises nominal rates to combat cost-push inflation, driving real interest rates higher. This precipitates a contractionary adjustment where investment, consumption, and output contracts. This trajectory serves as the baseline for a transition driven purely by carbon pricing without monetary accommodation. In contrast, the optimal rules reveal a spectrum of accommodation.

The standard Taylor rules and the loss-minimizing augmented rule effectively mimic the traditional benchmark but with slightly dampened recessionary effects. By tolerating a modest increase in inflation, these rules avoid the sharpest contraction in output seen under the strict traditional rule. However, the overall dynamic remains recessionary: investment stagnates, and the transition proceeds through a contraction in aggregate demand that facilitates the reallocation of resources away from the brown sector. A different adjustment path emerges under the welfare-maximizing augmented rule (solid orange). By responding positively to the decline in emissions ($\phi_e > 0$), this policy effectively lowers the real interest rate despite rising inflation. This monetary accommodation triggers a massive, front-loaded expansion in investment and a temporary boom in output. Consumption falls due to intertemporal substitution, but the drop is less pronounced than under the other rules. This "Green Boom" comes at a steep nominal cost: inflation spikes to over 4% (annualized) and nominal rates exhibit extreme volatility. While theoretically optimal for household welfare in this model, such aggressive accommodation generates nominal instability that would

likely be viewed as unfeasible or outside the mandate of a conventional inflation-targeting central bank.

Figure 1: Impulse Responses to Surprise Carbon Tax Shock

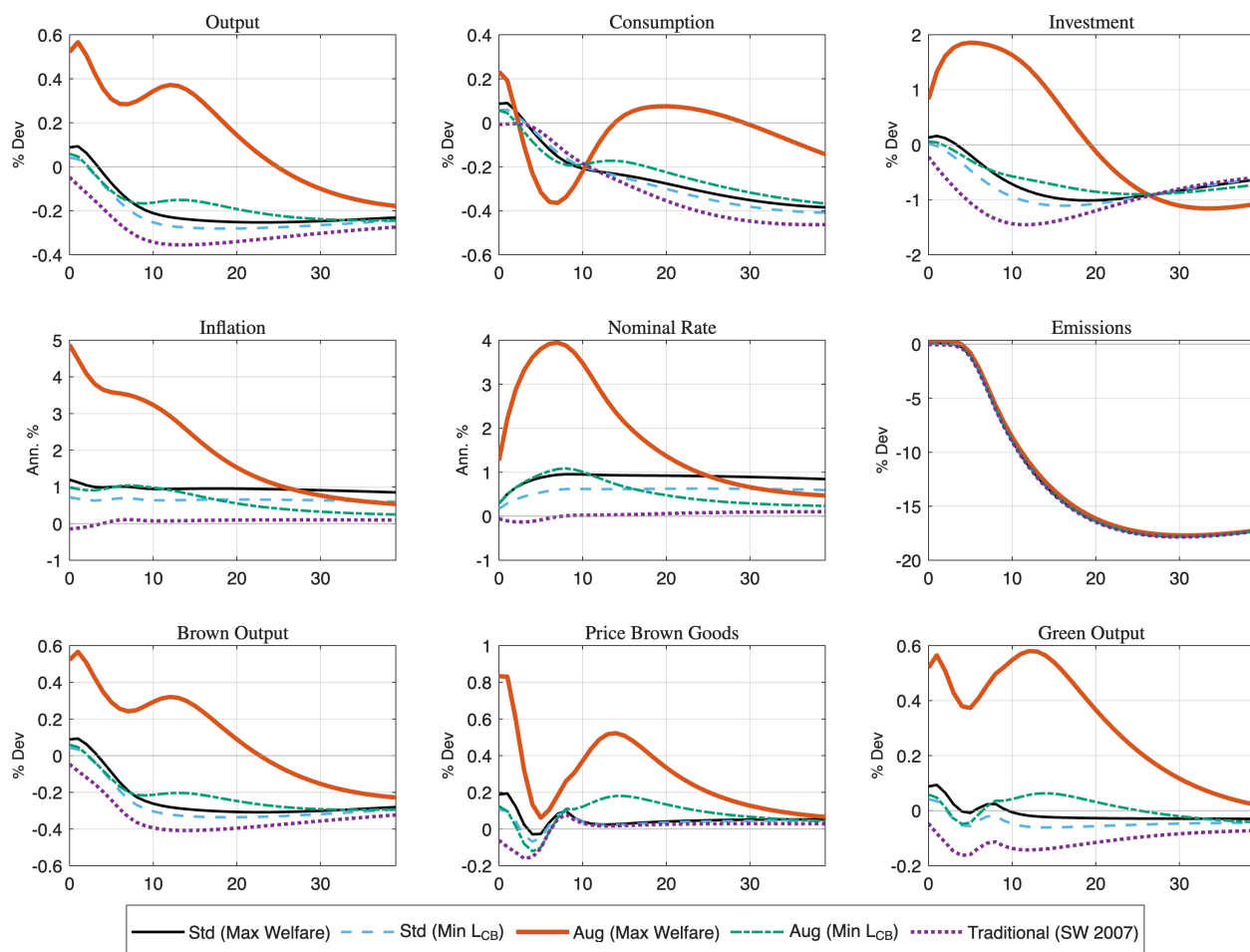


Impulse responses to a surprise increase in the carbon tax under five policy rules. Output, consumption, investment, emissions, brown output, and green output are expressed as percent deviations from baseline. Inflation and nominal rate are expressed in annualized percentage points. Horizontal axis shows quarters after the shock.

The financial transmission channel, presented in Appendix B, clarifies the mechanism behind these divergent paths. Under strict policy rules (Traditional and Loss-Minimizing), high real interest rates depress asset prices, causing a deterioration in bank balance sheets ("Financial Crunch") that amplifies the recession. Conversely, the welfare-maximizing augmented rule generates a "Financial Shielding" effect. The fall in real interest rates triggers an appreciation in green asset prices, boosting bank net worth and lowering credit spreads. This financial easing creates a virtuous cycle that supports the investment boom, demonstrating that monetary policy can actively use the financial accelerator to speed up the green transition, albeit at the cost of nominal instability.

Comparing shock structures, the anticipation of the tax (Figure 2) smooths the adjustment but does not alter the ranking of policies. The gradual phase-in allows forward-looking agents to adjust capital stocks earlier, mitigating the initial drop in consumption. However, the fundamental trade-off remains: strict adherence to price stability (Traditional/Loss-Minimizing) enforces a recessionary transition, while aggressive emissions-targeting (Welfare-Maximizing) can engineer an expansionary transition by tolerating high inflation volatility.

Figure 2: Impulse Responses to Gradual (Anticipated) Carbon Tax Shock



Impulse responses to a gradual carbon tax increase announced at $t = 0$ and phased in over quarters 4–8. Policy rules and variable definitions as in Figure 1. Horizontal axis shows quarters after the shock.

These impulse responses indicate that emissions-augmented monetary policy primarily reallocates adjustment costs over time rather than altering the long-run emissions path, which is largely determined by the carbon tax. Aggressive augmentation operates by accommodating the transition through an inflationary investment boom, shifting volatility away from real activity toward nominal

variables. The ranking of policy rules is stable across surprise and anticipated shocks, suggesting that the central bank’s optimal response depends more on its tolerance for nominal volatility than on the timing of the transition. The inflation dynamics under welfare maximization should therefore be interpreted as an upper bound on feasible accommodation, rather than as a realistic policy path.

The expansionary response observed under aggressive accommodation reflects the dominance of the monetary policy channel over the carbon tax’s direct cost-push effects. While the tax itself is contractionary, the strong easing induced by falling emissions more than offsets this effect, underscoring that the macroeconomic impact of carbon policy is highly sensitive to the monetary regime under which it is implemented.

5.4 Financial Amplification Channel

To assess the role of financial frictions in shaping the transition dynamics, Appendix A.3 reports impulse responses under an alternative specification in which banking frictions are shut down by eliminating credit spreads and leverage adjustment costs. In this frictionless environment, the qualitative ranking of policy rules is preserved, but the transmission mechanism and the quantitative differences across rules change substantially.

The key difference is in investment dynamics. Under the welfare-maximizing augmented rule and a surprise carbon tax, peak investment rises by approximately 5 percent in the baseline model with financial frictions, compared to less than 2 percent in the frictionless economy. This nearly three-fold amplification reflects the financial accelerator at work: by lowering real interest rates, the augmented rule supports Tobin’s Q , strengthens bank net worth, and compresses credit spreads, generating a feedback loop between asset valuations and lending capacity. Without this loop, monetary easing still stimulates investment through standard intertemporal substitution, but the response is substantially weaker. Output responses are correspondingly attenuated, though the difference is more moderate than for investment.

Interestingly, the attenuation is not uniform across variables. Consumption under the welfare-maximizing rule is somewhat larger in the frictionless economy, as the absence of intermediation frictions allows more of the monetary easing to pass through to household spending rather than being absorbed by bank balance-sheet repair. Similarly, the inflation response is not attenuated, if anything, it is marginally larger without financial frictions. This asymmetry suggests that financial

frictions redirect the effects of monetary accommodation toward investment rather than prices, rather than simply amplifying all responses uniformly.

Emissions dynamics are nearly identical across the two environments, confirming that the transition path is driven primarily by the carbon tax and abatement technology rather than by financial conditions. Monetary policy affects emissions only indirectly through aggregate activity, and the similarity of emissions paths reinforces the interpretation that the augmented rule responds to transition-induced macroeconomic stress rather than mechanically targeting environmental outcomes.

These results clarify the state-contingent nature of emissions-augmented monetary policy. The value of augmentation is greatest when transition shocks interact with financial frictions to generate balance-sheet stress and credit tightening—precisely the conditions under which conventional stabilization rules leave the most welfare on the table. In the frictionless economy, the gap between welfare-maximizing and loss-minimizing policies narrows substantially: without the financial accelerator, aggressive accommodation delivers smaller real gains while still generating comparable inflation volatility, making the welfare-stabilization trade-off less acute. Financial frictions are therefore important not only to the strength of monetary transmission during the transition, but also to the practical tension between welfare-oriented and mandate-consistent policy design.

5.5 Robustness checks - Alternative Parametrization

To assess whether the divergence between welfare-maximizing and stabilization-oriented monetary policy reflects a genuine economic trade-off rather than a feature of the baseline calibration, Table 7 reports optimal policy coefficients (ϕ_π^* , ϕ_e^*) across a broad set of alternative specifications for the surprise shock (Appendix A.1 presents the same table for the gradual shock). The robustness analysis spans variation in household preferences, structural rigidities, production technologies, and the size of the carbon tax shock.

The main conclusion is that the qualitative policy trade-off identified in the baseline is highly stable. Across changes in risk aversion (σ), intertemporal elasticity of substitution (ψ), habit persistence (ρ_c), and the elasticity of substitution across inputs (ξ), the ranking of policy regimes remains essentially unchanged. When welfare is the objective, the optimal rule consistently features a sizable positive response to emissions, with ϕ_e^* clustered around 0.10. In this regime, monetary

policy exploits the financial accelerator to generate an investment-driven expansion that cushions the real effects of the transition. By contrast, when the objective is to minimize the central bank loss, the optimal emissions response remains small, typically around $\phi_e^* \approx 0.025$, reflecting a strong preference for limiting nominal volatility. These patterns indicate that the tension between real smoothing and price stability is structural rather than parameter-driven.

Table 6: Robustness of Optimal Taylor Rule Parameters

Parameter	Max Welfare			Min L_{CB}		
	ϕ_π^*	ϕ_e^*	CE (%)	ϕ_π^*	ϕ_e^*	L_{CB}
<i>Risk Aversion (σ)</i>						
$\sigma = 2$	1.50	0.100	-0.089	1.10	0.025	0.0952
$\sigma = 10$ (baseline)	1.50	0.100	-0.084	1.10	0.025	0.0952
$\sigma = 30$	1.50	0.075	-0.091	1.10	0.025	0.0952
<i>Intertemporal Elasticity of Substitution (ψ)</i>						
$\psi = 0.5$	1.50	0.100	-0.092	1.10	0.025	0.108
$\psi = 1.5$ (baseline)	1.50	0.100	-0.084	1.10	0.025	0.0952
$\psi = 2.0$	1.50	0.100	-0.089	1.10	0.025	0.0936
$\psi = 5.0$	1.50	0.100	-0.088	1.10	0.025	0.0911
<i>Carbon Tax Shock Size</i>						
0.5%	1.50	0.200	-0.030	1.10	0.025	0.098
1.0% (baseline)	1.50	0.100	-0.084	1.10	0.025	0.091
2.0%	1.10	0.050	-0.220	1.10	0.025	0.093
5.0%	1.50	0.025	-0.762	1.50	0.025	0.099
10.0%	1.50	0.000	-2.127	1.50	0.025	0.102
<i>Habit Persistence (ρ_c)</i>						
$\rho_c = 0.0$	1.50	0.100	-0.087	1.10	0.025	0.097
$\rho_c = 0.3$	1.50	0.100	-0.087	1.10	0.025	0.096
$\rho_c = 0.7$ (baseline)	1.50	0.100	-0.084	1.10	0.025	0.095
$\rho_c = 0.8$	1.50	0.100	-0.090	1.10	0.025	0.095
<i>Abatement Technology</i>						
No Abatement	1.50	0.200	-0.067	1.50	0.200	0.196
With Abatement (baseline)	1.50	0.100	-0.084	1.10	0.025	0.095
<i>CRRA Preferences ($\psi = 1/\sigma$)</i>						
$\sigma = 2$	1.50	0.100	-0.092	1.10	0.025	0.108
$\sigma = 5$	1.50	0.075	-0.100	1.10	0.025	0.142
$\sigma = 10$	1.50	0.075	-0.109	1.50	0.025	0.194
<i>Elasticity of Substitution (ξ)</i>						
$\xi = 1.01$	1.50	0.100	-0.086	1.10	0.025	0.096
$\xi = 2.0$ (baseline)	1.50	0.100	-0.084	1.10	0.025	0.095

Notes: Optimal (ϕ_π, ϕ_e) selected over the grid $\phi_\pi \in \{1.10, 1.50\}$ and $\phi_e \in [-0.10, 0.15]$, holding $\phi_y = 0.125$ fixed. CE denotes consumption-equivalent welfare. L_{CB} is the cumulative discounted central bank loss. Results based on 2,000 order-2 GIRF replications with pruning.

The importance of technological constraints is illustrated by the no-abatement scenario. When firms lack access to abatement technologies and can only respond to carbon pricing by cutting

output, the transition shock becomes significantly more contractionary. In this case, the optimal emissions coefficient rises sharply for both objectives, reaching $\phi_e^* = 0.20$. The central bank responds by providing aggressive financial accommodation to offset the lack of technological adjustment margins and prevent a collapse in aggregate demand. This result highlights that the scope for monetary stabilization expands when the real economy is less flexible, even though such policies may come at a higher inflationary cost.

The exercise also reveal a pronounced non-linearity with respect to the size of the carbon tax shock. For relatively small shocks, monetary policy can afford to respond aggressively to emissions dynamics, effectively smoothing the transition through accommodative financial conditions without destabilizing inflation. As the magnitude of the shock increases, however, the optimal emissions response declines rapidly and eventually converges toward zero. Large carbon tax increases generate substantial cost-push inflation on their own, leaving little room for additional accommodation. In such environments, a positive emissions response would amplify inflationary pressures to an extent that is inconsistent with price stability. This non-linearity underscores a fundamental limit to the role of monetary policy in managing the transition: when the shock itself threatens nominal stability, the central bank must revert to orthodox inflation control.

Finally, comparing Epstein–Zin preferences to standard CRRA utility clarifies the role of risk sensitivity in shaping optimal policy. Under CRRA preferences with high risk aversion, the loss-minimizing policy places substantially greater weight on inflation stabilization, with ϕ_π^* rising to 1.50 relative to the baseline Epstein–Zin specification. This reflects the reduced ability of CRRA preferences to disentangle risk aversion from intertemporal substitution, which increases the welfare cost of nominal volatility. The comparison shows the benefit of the Epstein–Zin framework for analyzing transition shocks, in which short-run risk and long-run adjustment incentives interact.

In Appendix A.5 I present the Impulse Response functions for CRRA preferences, underscoring that Epstein-Zin preferences are essential for isolating the distinct roles of risk aversion and intertemporal substitution. Comparing the baseline EZ specification against CRRA alternatives reveals that the *elasticity of intertemporal substitution*, rather than risk aversion, governs policy transmission. Under the welfare-maximizing rule, high-IES specifications generate a robust investment boom (rising 2.5% under EZ vs. 0.5% under low-IES CRRA) and stronger financial accelerator effects, characterized by deeper spread compression and higher asset valuations. Conversely, when

the IES is low, monetary accommodation leaks primarily into prices rather than quantities; peak inflation rises from 4.2% (EZ) to 4.8% (CRRA), worsening the stabilization trade-off. Consequently, the choice of preference specification is not merely technical; it determines the transmission mechanism: EZ activates a quantity-driven investment channel, whereas low-IES CRRA activates a price-driven consumption-smoothing channel.

function with $\sigma = 10$ and $\psi = 1/10$. Under CRRA with low ies, households are reluctant to substitute consumption across time: the investment boom that characterizes the welfare-maximizing augmented rule under EZ preferences essentially vanishes under CRRA. Peak investment rises by approximately 2.5% under EZ but only 0.5% under CRRA—an 80% attenuation. This occurs because the low IES prevents households from accepting the temporary consumption decline needed to finance the investment surge. Second, the financial transmission channel operates differently. Credit spreads compress by 50 basis points under EZ but only 30 basis points under CRRA, as the weaker investment response reduces the demand for credit and the associated improvement in bank balance sheets. This distinction matters for policy: under CRRA, aggressive accommodation generates larger inflation volatility relative to the real gains achieved, explaining why the loss-minimizing rule assigns a higher inflation coefficient ($\phi_\pi^* = 1.50$) when IES is low.

5.6 Robustness - Monetary Policy Shocks

To assess whether emissions augmentation distorts standard monetary transmission, I examine the economy’s response to an exogenous contractionary monetary policy shock under the standard, emissions-augmented, and traditional Taylor rules in Appendix A.4. Across all specifications, real responses are closely aligned. Output, consumption, and investment fall on impact and recover gradually, while inflation declines and the nominal interest rate rises temporarily. These dynamics are consistent with conventional New Keynesian transmission.

Allowing the policy rule to respond to emissions does not materially alter the real effects of monetary disturbances. Differences across rules are confined to nominal variables and reflect how the policy rule internalizes emissions dynamics rather than changes in the underlying demand-side transmission mechanism. In particular, emissions augmentation does not introduce amplification or attenuation of real activity in response to monetary shocks. This confirms that augmenting the policy rule with emissions preserves the standard stabilization properties of Taylor-rule-based

monetary policy.

This result is important for interpretation: emissions augmentation affects the conduct of policy during transition shocks without compromising the central bank’s ability to manage conventional monetary disturbances.

6 Discussion and Policy Implications

The results in Section 5 highlight a fundamental tension in the conduct of monetary policy during a green transition: the policy that maximizes household welfare is not compatible with the stabilization mandates of modern independent central banks. This tension is not quantitative but structural, shaping how emissions-augmented rules should be interpreted and implemented in practice.

The analysis focuses on the interest rate channel, which transmits transition shocks to the financial sector through asset valuations, collateral constraints, and bank net worth. While central banks also possess balance sheet tools that could target spreads more directly, existing evidence suggests that these instruments are limited by market depth and institutional constraints (Nakov and Thomas, 2023). By contrast, the financial shielding mechanism identified here shows that conventional interest rate policy can already relax financial conditions during the transition, conditional on the central bank’s tolerance for inflation volatility.

6.1 The Infeasibility of the Welfare-Optimal Strategy

From a theoretical perspective, the welfare-maximizing augmented rule delivers the largest consumption-equivalent gains by engineering an investment boom that cushions the economy against the carbon tax. However, the mechanism required to achieve these gains renders the policy infeasible for three reasons.

First, the strategy relies on substantial inflation accommodation. As shown in Figure 1, the welfare-optimal rule generates inflation rates well above standard targets, pushing real interest rates into negative territory. While this supports capital accumulation, it directly conflicts with price stability mandates.

Second, the policy induces excessive nominal volatility. Relative to the standard rule, the welfare-maximizing strategy increases the central bank loss by several hundred percent (Table 5). Such

volatility would likely undermine credibility and weaken the central bank’s ability to stabilize future shocks.

Third, the resulting demand expansion partially offsets the emissions-reducing intent of the carbon tax. As reported in Table 4, cumulative emissions fall by less under the welfare-maximizing rule than under the standard benchmark. A policy that maximizes welfare through monetary expansion therefore risks diluting the effectiveness of fiscal climate policy.

6.2 A Pragmatic Approach: Emissions as a Supply-Side Indicator

In contrast, the loss-minimizing results provide a feasible, mandate-consistent policy benchmark. When the objective is stabilization, the optimal response to emissions is positive but small ($\phi_e \approx 0.025$). Emissions dynamics are best interpreted not as a policy target, but as an informative indicator of transition-related supply pressures.

Because emissions move closely with the contraction of brown production following the carbon tax, they provide timely information about the severity of the supply shock. A modest response to emissions allows the central bank to fine-tune the inflation–output trade-off without triggering the destabilizing boom-bust dynamics associated with welfare maximization. This pragmatic augmentation improves stabilization outcomes without materially affecting the emissions path determined by fiscal policy.

6.3 The Division between Fiscal and Monetary Policy

These findings reinforce a clear division of responsibilities. The long-run trajectory of emissions is overwhelmingly determined by fiscal policy, with monetary policy having only a marginal effect on cumulative emissions. The optimal role of the central bank is therefore not to substitute for climate regulation, nor to engineer a welfare-maximizing boom that violates its mandate.

Instead, a feasible optimal policy combines a strong response to inflation with limited, state-contingent flexibility based on emissions data to smooth the transition-induced reallocation. By respecting this division, monetary policy anchors nominal expectations while allowing fiscal policy to guide the economy’s structural transformation through relative price signals. In this sense, the most effective “green” monetary policy is one that preserves nominal stability and supports long-term investment during the transition.

7 Conclusions

This paper studies how monetary policy should respond to the low-carbon transition when climate policy operates through carbon taxation and financial frictions shape macroeconomic adjustment. Using a two-sector E-DSGE model with banking constraints, endogenous abatement, and recursive preferences, the analysis shows that the transition is not only a supply shock but also a financial shock, with asset revaluations and balance-sheet dynamics playing a central role in transmission.

The results highlight a sharp distinction between welfare-optimal and mandate-consistent monetary policy. Rules that aggressively accommodate declining emissions can generate meaningful welfare gains by smoothing consumption risk and supporting investment through the financial accelerator. These gains arise from reallocating adjustment costs intertemporally rather than from altering the long-run emissions path, which remains pinned down by fiscal policy. However, the same mechanism produces large and persistent inflation volatility, rendering such policies incompatible with standard inflation-targeting mandates and central bank credibility.

When policy is instead evaluated through a conventional stabilization lens, the optimal response changes markedly. A small, positive reaction to emissions improves outcomes modestly without destabilizing prices or output, while preserving the standard transmission of monetary policy to demand. In this regime, emissions act as an auxiliary indicator of transition-related supply pressures rather than as an independent policy objective. Importantly, the ranking of policy rules is robust across surprise and anticipated tax implementations, across parameterizations, and to removing financial frictions, though the quantitative importance of accommodation is substantially reduced when the financial accelerator is absent.

Together, the findings support a division of roles. Fiscal policy determines the path of decarbonization through carbon pricing, while monetary policy handles how the economy absorbs the resulting effects. Central banks need not ignore the transition, but neither should they attempt to drive it directly. An approach that treats emissions dynamics as a state-contingent signal of supply stress allows monetary policy to smooth adjustment while maintaining nominal stability, ensuring that the green transition proceeds without compromising central bank's mandate. Hence, their role is not to substitute for climate policy, but to ensure that the transition is not derailed by financial instability.

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Appendix A.1 Alternative Parametrization - Gradual Tax Shock

Table 7: Robustness of Optimal Taylor Rule Parameters - Gradual Tax

Parameter	Max Welfare			Min L_{CB}		
	ϕ_π^*	ϕ_e^*	CE (%)	ϕ_π^*	ϕ_e^*	L_{CB}
<i>Risk Aversion (σ)</i>						
$\sigma = 2$	1.10	0.050	-0.094	1.10	0.025	0.095
$\sigma = 10$ (baseline)	1.10	0.050	-0.094	1.10	0.025	0.095
$\sigma = 30$	1.50	0.100	-0.092	1.10	0.025	0.095
<i>Intertemporal Elasticity of Substitution (ψ)</i>						
$\psi = 0.5$	1.10	0.050	-0.097	1.10	0.025	0.108
$\psi = 1.5$ (baseline)	1.10	0.050	-0.094	1.10	0.025	0.095
$\psi = 2.0$	1.10	0.050	-0.094	1.10	0.025	0.094
$\psi = 5.0$	1.10	0.050	-0.094	1.10	0.025	0.091
<i>Carbon Tax Shock Size</i>						
0.5%	1.10	0.100	-0.040	1.10	0.025	0.090
1.0% (baseline)	1.10	0.050	-0.094	1.10	0.025	0.095
2.0%	1.10	0.050	-0.228	1.10	0.025	0.093
5.0%	1.50	0.025	-0.759	1.50	0.025	0.095
10.0%	1.50	0.000	-2.051	1.50	0.025	0.120
<i>Habit Persistence (ρ_c)</i>						
$\rho_c = 0.0$	1.10	0.050	-0.093	1.10	0.025	0.097
$\rho_c = 0.3$	1.10	0.050	-0.093	1.10	0.025	0.096
$\rho_c = 0.7$ (baseline)	1.10	0.050	-0.094	1.10	0.025	0.095
$\rho_c = 0.8$	1.10	0.050	-0.095	1.10	0.025	0.095
<i>Abatement Technology</i>						
No Abatement	1.50	0.200	-0.069	1.50	0.200	0.20
With Abatement (baseline)	1.10	0.050	-0.094	1.10	0.025	0.095
<i>CRRA Preferences ($\psi = 1/\sigma$)</i>						
$\sigma = 2$	1.10	0.050	-0.097	1.10	0.025	0.108
$\sigma = 10$	1.10	0.050	-0.112	1.50	0.025	0.194
<i>Elasticity of Substitution (ξ)</i>						
$\xi = 1.01$	1.10	0.050	-0.092	1.10	0.025	0.096
$\xi = 2.0$ (baseline)	1.10	0.050	-0.094	1.10	0.025	0.095
$\xi = 3.00$	1.10	0.050	-0.096	1.10	0.025	0.094

Notes: Optimal (ϕ_π, ϕ_e) computed over the grid $\phi_\pi \in \{1.10, 1.50\}$ and $\phi_e \in [-0.10, 0.15]$, holding $\phi_y = 0.12$ fixed. Results based on 2000 GIRF replications with order=2 pruning. CE denotes consumption-equivalent welfare. L_{CB} is the cumulative discounted central bank loss.

Appendix A.2 Transition Dynamics for Financial Variables

Figure 3: Impulse Responses to Surprise Carbon Tax Shock

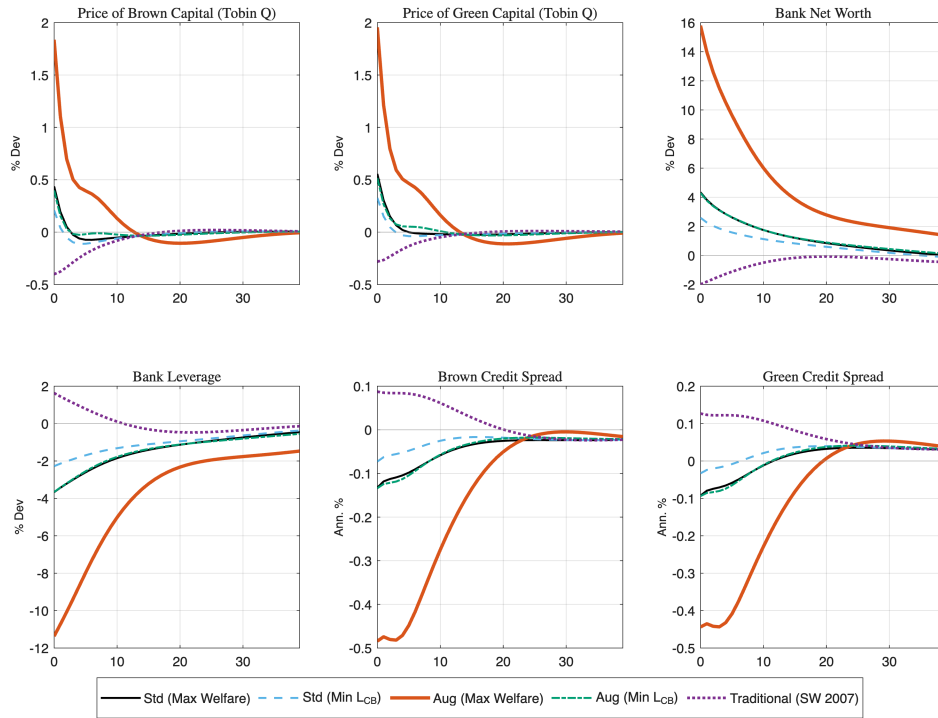
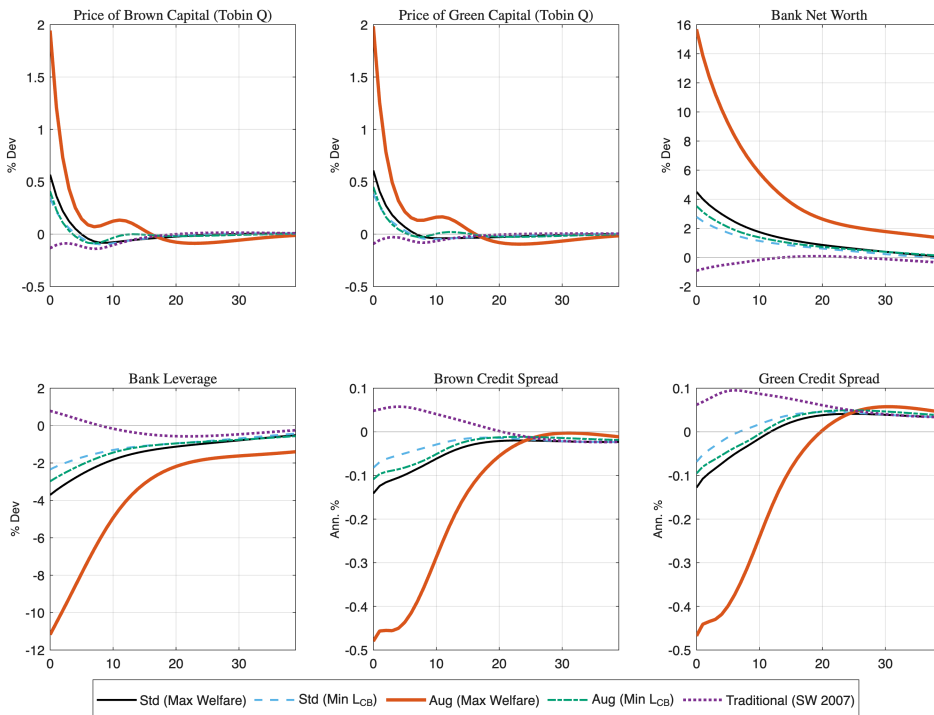


Figure 4: Impulse Responses to Gradual (Anticipated) Carbon Tax Shock



7.1 Appendix A.3: Transition Dynamics without Financial Frictions

Figure 5: Impulse Responses to Surprise Carbon Tax Shock - No Financial Frictions

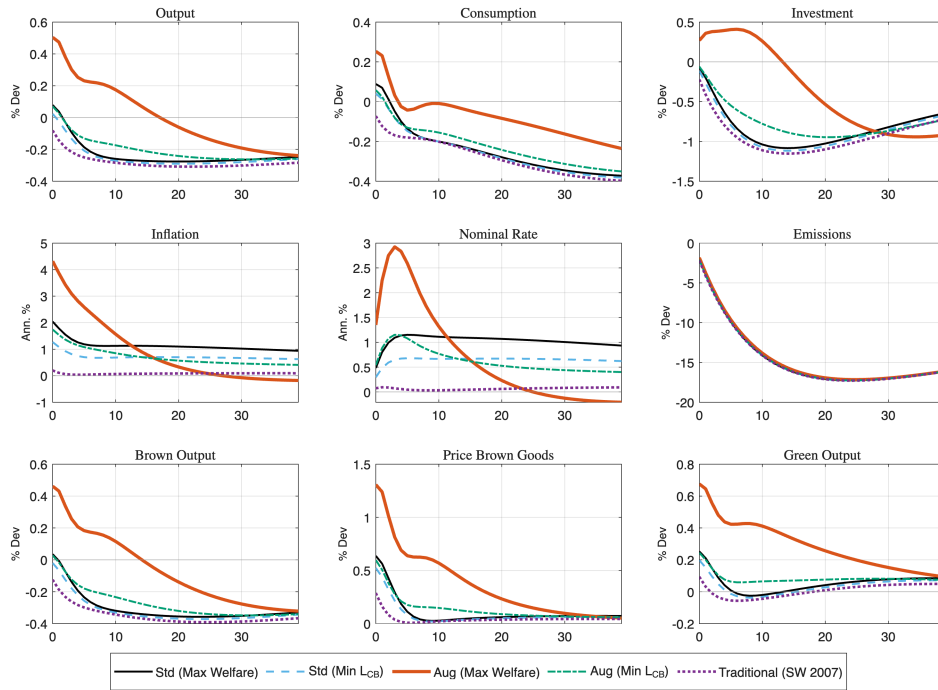
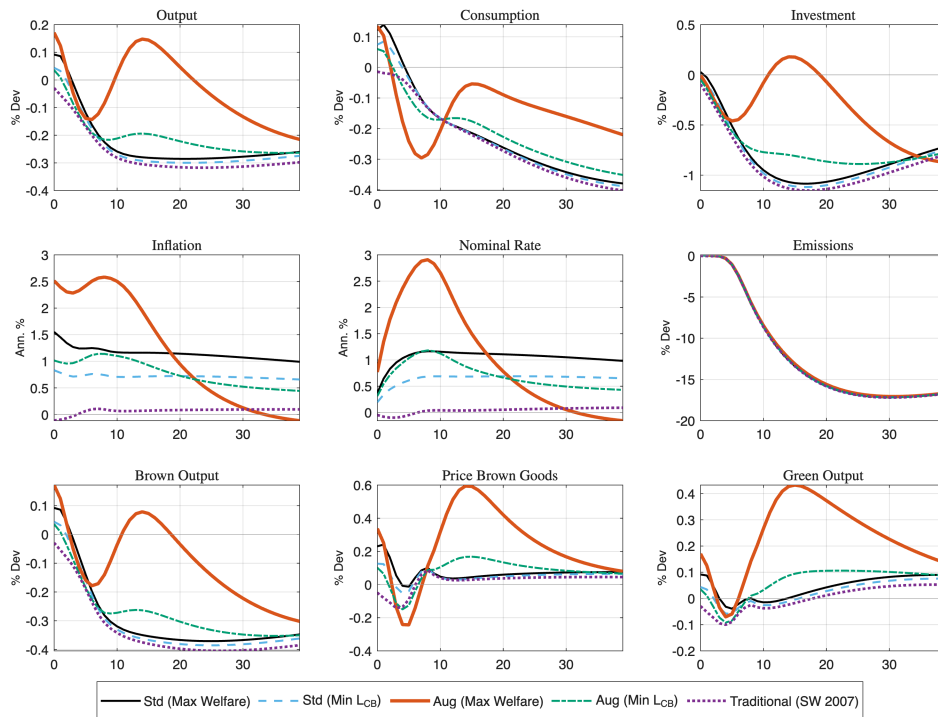


Figure 6: Impulse Responses to Gradual (Anticipated) Carbon Tax Shock - No Financial Frictions



7.2 Appendix A.4: Impulse Response functions to a 25 bp Monetary Policy Shock

Figure 7: Impulse Responses to Monetary Policy Shock Shock - Real Variables

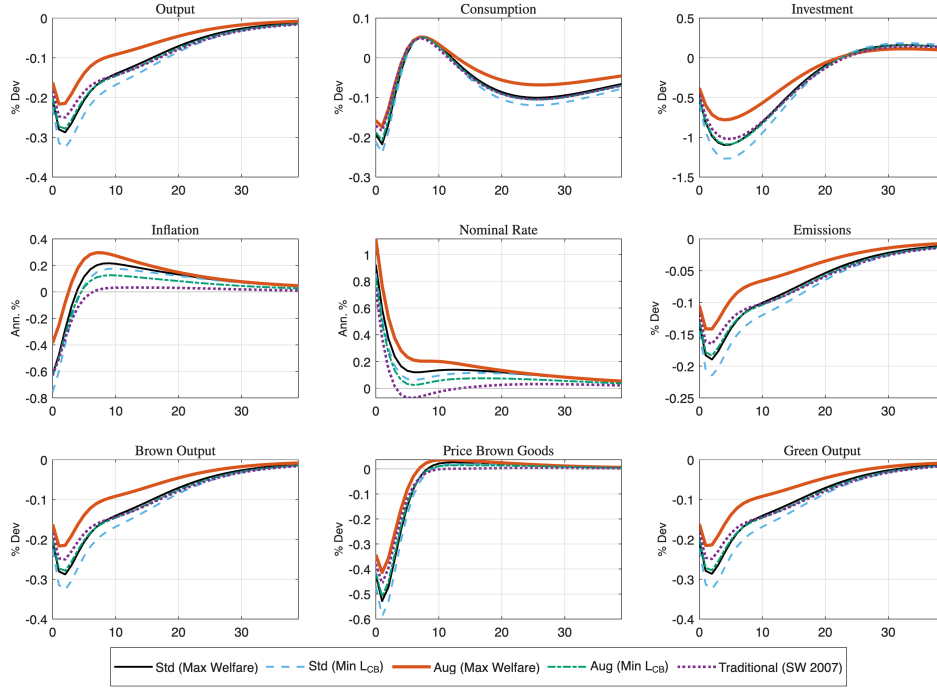
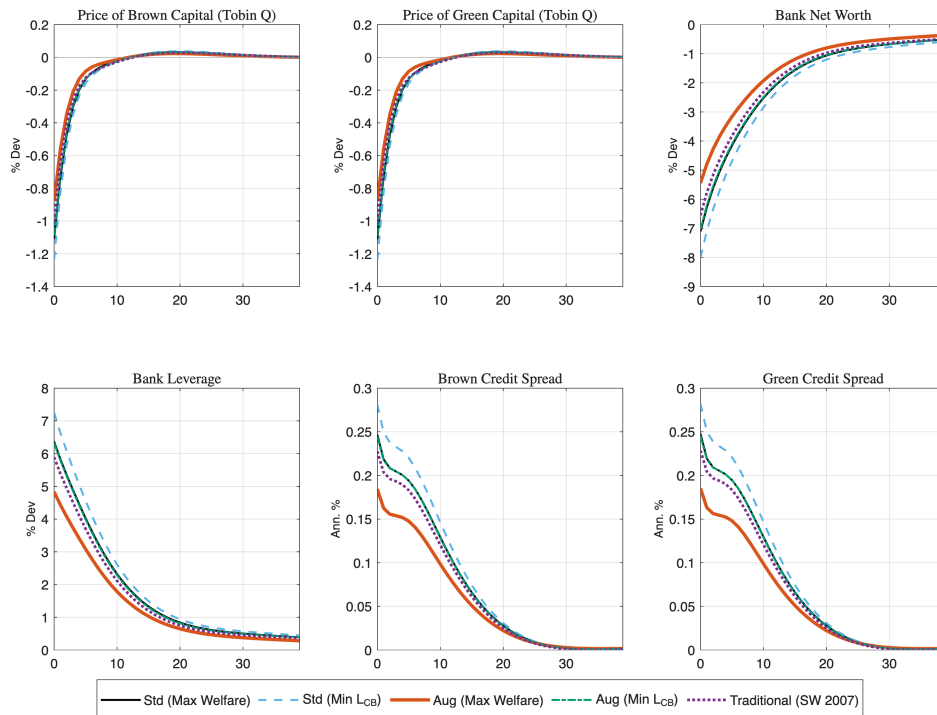


Figure 8: Impulse Responses to Monetary Policy Shock - Financial Variables



7.3 Appendix A.5: Impulse Response functions with CRRA preferences

Figure 9: Impulse Responses to Surprise Carbon Tax - Real Variables - CRRA $\sigma = 10$ and $\psi = 1/10$

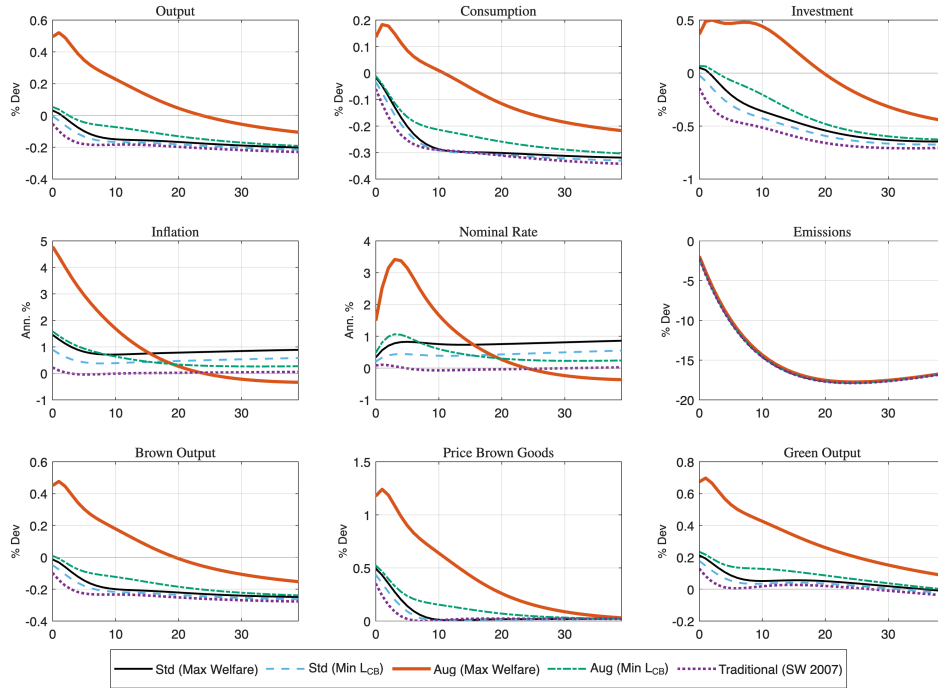


Figure 10: Impulse Responses to Surprise Carbon Tax - Financial Variables - CRRA $\sigma = 10$ and $\psi = 1/10$

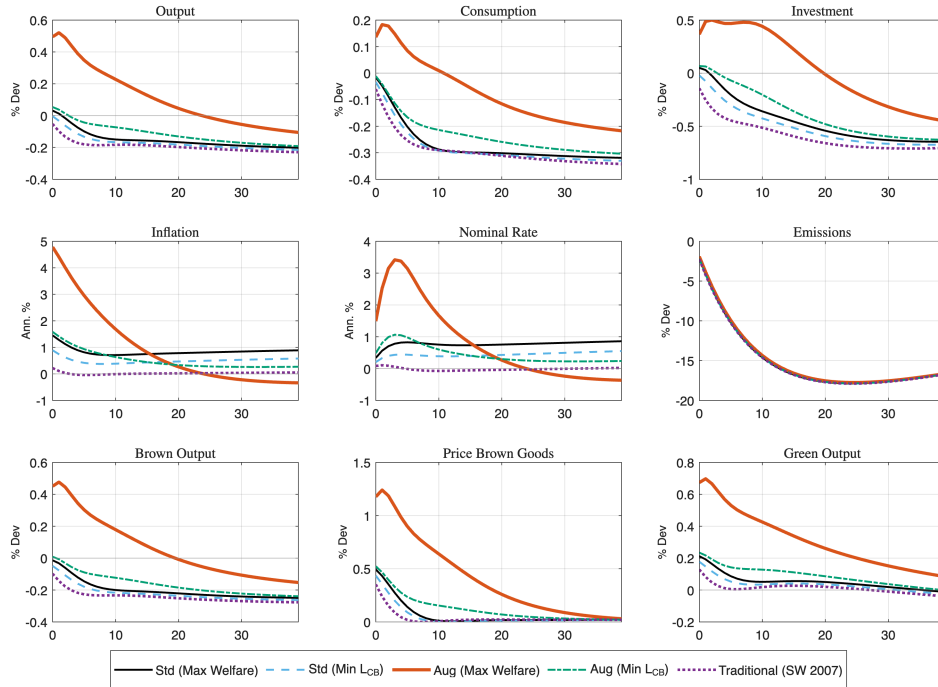


Figure 11: Impulse Responses to Surprise Carbon Tax - Real Variables - CRRA $\sigma = 2$ and $\psi = 1/2$

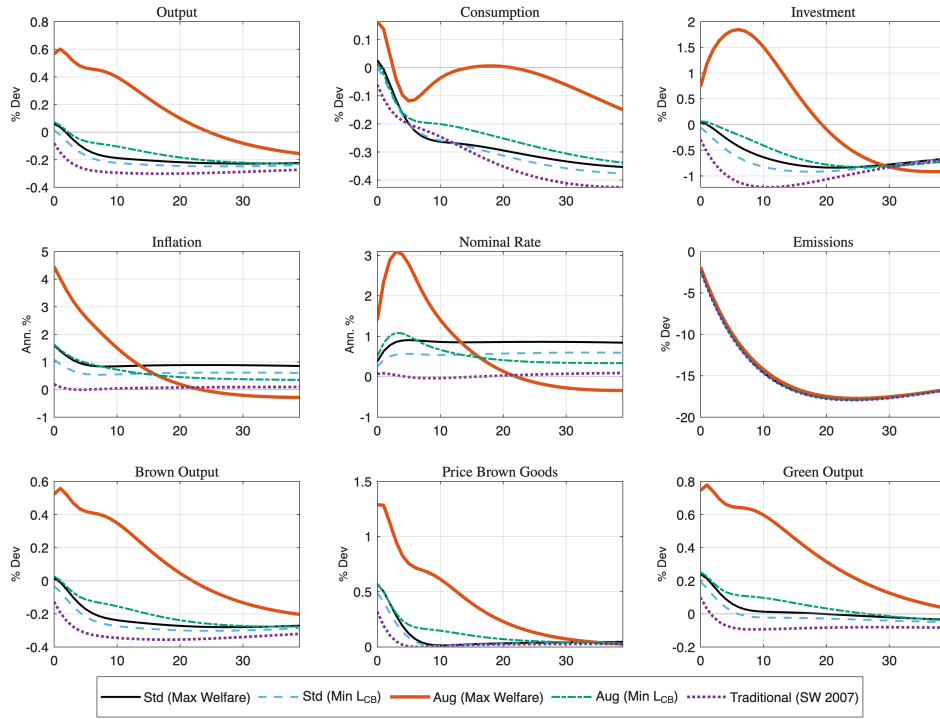
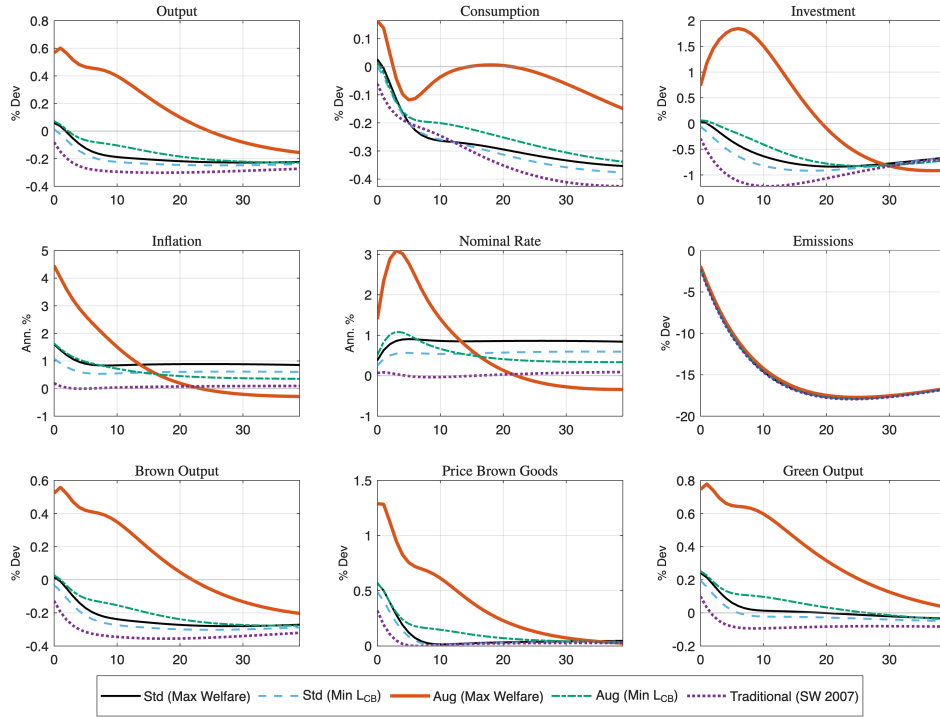


Figure 12: Impulse Responses to Surprise Carbon Tax - Financial Variables - CRRA $\sigma = 2$ and $\psi = 1/2$



Appendix B.1: Calibrated Parameters

We follow Ferrari and Nispi Landi (2024) and calibrate ex-ante the parameters:

- $Lev = 6$
- $\frac{\bar{r}}{\bar{p}i} = R = 1.05$
- $\frac{k^G}{k} = 0.2$

And we find the ex post parameters: $\{\theta, \iota, \zeta, g\}$

By using the net-worth evolution, we can find ι :

$$\iota = \frac{1 - \chi((r^G - R)lev + R)}{lev}$$

Then, from the FOC of bankers, we can get the bank's discount factor:

$$\nu = \frac{1 - \chi}{1 - \chi\beta((r^G - R)lev + R)}$$

And from the leverage equation, we get the fraction of capital that can be diverted:

$$\theta = \frac{\nu}{lev} + \beta((r^G - R)\nu)$$

In the model, pollution is represented in arbitrary units. However, to align with real-world measurements, the aim is to calibrate the model so that the steady-state pollution level corresponds to x^{GtC} (gigatons of carbon). This choice of x^{GtC} will result in a damage parameter, denoted as \bar{D} , that matches the estimate provided by Gibson and Heutel (2023). By achieving this calibration, I ensure the model's pollution levels are more meaningful and relevant to actual carbon emissions and their associated impacts.

$$\bar{D} = \bar{D}_0 + \bar{D}_1 x^{GtC} + \bar{D}_2 x^{GtC^2}$$

Where \bar{D}_1 and \bar{D}_2 are proposed by Gibson and Heutel (2023). We need to find D_1 and D_2 in the model. Define the emissions of the rest of the world as: $e^{row} = RoW \times \bar{e}$, and from the steady-state pollution stock (\bar{x}). Then we get:

$$D_1 = \overline{D}_1 \frac{x^{Gtc}}{\bar{x}}, \quad D_2 = \overline{D}_2 \left(\frac{x^{Gtc}}{\bar{x}} \right)$$

Appendix B.2: Steady State Equilibrium

This appendix details the system of equations characterizing the deterministic steady state of the model. In the steady state, all aggregate shocks are zero ($A_t = 1, \epsilon_t^\tau = 0$), and variables are constant. We denote steady-state values with a bar (e.g., \bar{Y}).

C.1 Prices and Interest Rates

The steady state is characterized by constant inflation $\bar{\pi}$. The real marginal cost \bar{p}_I is determined by the monopolistic competition markup ϵ :

$$\bar{p}_I = \frac{\epsilon - 1}{\epsilon} \quad (32)$$

The nominal interest rate \bar{R} is determined by the household's discount factor β and the inflation target according to the Fisher equation:

$$\bar{R} = \frac{\bar{\pi}}{\beta} \quad (33)$$

Due to financial frictions, there is a spread ($\bar{s}\bar{p}$) between the risk-free rate and the return on corporate bonds. The required returns on green and brown bonds are symmetric in the steady state:

$$\bar{R}^G = \bar{R}^B = \frac{1}{\beta} + \bar{s}\bar{p} \quad (34)$$

In the absence of investment adjustment costs ($\kappa_I = 0$ in steady state), the price of capital (Tobin's Q) is unity: $\bar{Q}^G = \bar{Q}^B = 1$. The rental rates of capital are thus determined by the required return and the depreciation rate δ :

$$\bar{R}_G^K = \bar{R}^G - (1 - \delta), \quad \bar{R}_B^K = \bar{R}^B - (1 - \delta) \quad (35)$$

C.2 Optimal Abatement and Emissions

A key feature of the model is endogenous abatement. Brown firms choose a steady-state abatement rate $\bar{\mu}$ to equate the marginal cost of abatement technology to the marginal tax savings. Given the carbon tax $\bar{\tau}$ and abatement cost parameters θ_1, θ_2 :

$$\underbrace{\theta_1 \theta_2 \bar{\mu}^{\theta_2 - 1}}_{\text{Marginal Cost}} = \underbrace{\bar{\tau} (\bar{Y}^B)^{-ps}}_{\text{Marginal Benefit}} \quad (36)$$

Solving for $\bar{\mu}$:

$$\bar{\mu} = \left(\frac{\bar{\tau} (\bar{Y}^B)^{-ps}}{\theta_1 \theta_2} \right)^{\frac{1}{\theta_2 - 1}} \quad (37)$$

Total real abatement costs \bar{Z} and net emissions \bar{E} are given by:

$$\bar{Z} = \theta_1 \bar{\mu}^{\theta_2} \bar{Y}^B, \quad \bar{E} = (1 - \bar{\mu}) (\bar{Y}^B)^{1-ps} \quad (38)$$

C.3 Production and Factor Demands

Output is aggregated from green and brown sectors using a CES technology with elasticity of substitution ξ and weight ζ . The relative prices satisfy the aggregate price index condition:

$$1 = (1 - \zeta) \left(\frac{\bar{p}_G}{\bar{p}_I} \right)^{1-\xi} + \zeta \left(\frac{\bar{p}_B}{\bar{p}_I} \right)^{1-\xi} \quad (39)$$

This implies the relative demand for sectoral output:

$$\bar{Y}^G = (1 - \zeta) \left(\frac{\bar{p}_G}{\bar{p}_I} \right)^{-\xi} \bar{Y}, \quad \bar{Y}^B = \zeta \left(\frac{\bar{p}_B}{\bar{p}_I} \right)^{-\xi} \bar{Y} \quad (40)$$

In the Green Sector, factor demands follow standard Cobb-Douglas conditions:

$$\bar{w} = (1 - \alpha) \bar{p}_G \frac{\bar{Y}^G}{\bar{L}^G}, \quad \bar{R}_G^K = \alpha \bar{p}_G \frac{\bar{Y}^G}{\bar{K}^G} \quad (41)$$

In the Brown Sector, the carbon tax and abatement costs act as a wedge on revenue. Firms pay factors from their effective price \bar{p}_B^{eff} , defined as the market price net of abatement costs and tax payments:

$$\bar{p}_B^{eff} = \bar{p}_B - \frac{\bar{Z}}{\bar{Y}^B} - \bar{\tau}(1 - \bar{\mu})(1 - ps)(\bar{Y}^B)^{-ps} \quad (42)$$

The brown factor demands are thus:

$$\bar{w} = (1 - \alpha)\bar{p}_B^{eff} \frac{\bar{Y}^B}{\bar{L}^B}, \quad \bar{r}_B^K = \alpha\bar{p}_B^{eff} \frac{\bar{Y}^B}{\bar{K}^B} \quad (43)$$

C.4 Financial Sector

Financial intermediaries face an agency friction that limits leverage. In the steady state, the incentive constraint binds, linking the bank's value \bar{V}^f to its assets. Net worth \bar{N} evolves according to the accumulation of retained earnings from the spread between asset returns and deposit costs:

$$\bar{N} = \chi (\bar{R}^B \bar{\phi} - \bar{R}(\bar{\phi} - 1)) \bar{N} + \iota \bar{B} \quad (44)$$

where $\bar{\phi}$ is the steady-state leverage ratio, χ is the survival probability, and ι represents transfers to new bankers. The stochastic discount factor of the banker $\bar{\nu}$ is determined by:

$$\bar{\nu} = \frac{1 - \chi}{1 - \chi\beta[(\bar{R}^B - \bar{R})\bar{\phi} + \bar{R}]} \quad (45)$$

The diversion parameter θ is calibrated to ensure the incentive constraint holds at the target leverage $\bar{\phi}$:

$$\theta = \frac{\bar{\nu}}{\bar{\phi}} + \beta\bar{\nu}(\bar{R}^B - \bar{R}) \quad (46)$$

C.5 Market Clearing

Labor markets clear such that aggregate labor \bar{L} aggregates sectoral hours:

$$\bar{L} = [(\bar{L}^G)^{1+\rho_l} + (\bar{L}^B)^{1+\rho_l}]^{\frac{1}{1+\rho_l}} \quad (47)$$

The aggregate resource constraint accounts for consumption, investment, government spending, and the real resource costs of abatement:

$$\bar{Y} = \bar{C} + \bar{I} + \bar{G} + \bar{Z} \quad (48)$$

where aggregate investment covers depreciation of the total capital stock: $\bar{I} = \delta(\bar{K}^G + \bar{K}^B)$.

Appendix B.3: Banker's Problem Maximization

We solve this problem using the solution Ferrari and Nispi Landi (2024) proposed for this specific setting.

Let $M_{t,t+i}$ be the stochastic factor applying in period t to earnings at $t+i$. Each banker exits the market with probability $(1 - \chi)$ getting NW_{t+1} at the beginning of period $t + i$. With probability χ , a banker continues in this activity. Therefore the value of a bank is:

$$V_t^f = \max E_t \left[\sum_{i=0}^{\infty} (1 - \chi) \chi^i \beta^{i+1} M_{t,t+i} NW_{t+1+i} \right]$$

Subject to:

$$\frac{NW_t}{NW_{t-1}} = (r_t^G - r_t^B) lev_{t-1}^g + (r_t^B - \frac{r_{t-1}}{\pi_t}) lev_{t-1} + \frac{r_{t-1}}{\pi_t} - \frac{\kappa_F}{2} \left(\frac{lev_t^g}{l_{t-1}} - b^* \right)^2 \quad (49)$$

Recursively writing the expected terminal wealth:

$$\frac{V_t^f}{NW_t} = \max E_t \left[(1 - \chi) M_{t,t+1} \frac{NW_{t+1}}{NW_t} + \chi \beta M_{t,t+1} \frac{V_{t+1}^f}{NW_{t+1}} \right]$$

And following Gertler and Karadi (2011), financial intermediaries face the collateral constraint:

$$V_t^f(NW_t) \geq \theta b_t^f$$

$$\frac{V_t^f(NW_t)}{NW_t} \geq \theta lev_t$$

As the standard solution in this literature, we guess the solution:

$$\frac{V_t^f}{NW_t} = a_{gt}lev_t^g + a_{bt}lev_t + a_{ct}\left(\frac{lev_t^g}{lev_t} - b^*\right)^2 + a_{dt}$$

Being a_{kt} with $k = a, g, c, d$ the coefficients to be determined.

Rewriting the bank problem as:

$$\max \left[a_{gt}lev_t^g + a_{bt}lev_t + a_{ct}\left(\frac{lev_t^g}{lev_t} - b^*\right)^2 + a_{dt} \right] + (1 + \gamma_{Ft}) - \gamma_{Ft}\theta lev_t$$

With γ_{Ft} being the Lagrange multiplier. The FOC are:

$$\left[a_{gt} + \frac{2a_{ct}}{lev_t} \left(\frac{lev_t^g}{lev_t} - b^* \right) \right] (1 + \gamma_{Ft}) = 0, \quad \left[a_{bt} - \frac{2a_{ct}lev_t^g}{lev_t^2} \left(\frac{lev_t^g}{lev_t} - b^* \right) \right] (1 + \gamma_{Ft}) = \gamma_{Ft}\theta$$

Then, from the first FOC, we get the following:

$$a_{gt} = -\frac{2a_{ct}}{lev_t} \left(\frac{lev_t^g}{lev_t} - b^* \right)$$

We get the formula for γ_{Ft} from the second condition. Then, using the guessed solution, we get:

$$\frac{V_t^f}{NW_t} = \max E_t \left[(M_{t,t+1}\nu_{t+1} \frac{NW_{t+1}}{NW_t}) \right]$$

Where ν_t is the effective bank's SDF with $\nu_t = 1 - \chi + \chi \left[a_{bt} - \frac{2a_{ct}lev_t^g}{lev_t^2} \left(\frac{lev_t^g}{lev_t} - b^* \right)^2 \right]$

Given the Banks' constraint by the evolution of net worth, we can find the remaining coefficients:

$$a_{gt} = \mathbb{E}_t \left[M_{t,t+1}\nu_{t+1}(r_{t+1}^g - r_{t+1}^b) \right]$$

$$a_{bt} = \mathbb{E}_t \left[M_{t,t+1}\nu_{t+1}(r_{t+1}^b - \frac{r_t}{\pi_{t+1}}) \right]$$

$$a_{ct} = \mathbb{E}_t \left[M_{t,t+1}\nu_{t+1} \frac{r_t}{\pi_{t+1}} \right]$$

$$a_{dt} = -\frac{\kappa_F}{2} \mathbb{E}_t \left[M_{t,t+1} \nu_{t+1} \right]$$

Using the first FOC, we get the following:

$$E_t M_{t,t+1} \nu_{t+1} \left(r_{g,t+1} - r_{b,t+1} - \left(\frac{lev_{g,t}}{lev_t} - b^* \right) \frac{\kappa_F}{lev_t} \right) = 0$$

Now, we can get the respective expression for the optimal leverage:

$$\begin{aligned} \theta lev_t &= a_{gt} lev_t^g + a_{bt} lev_t + a_{ct} \left(\frac{lev_t^g}{lev_t} - b^* \right)^2 + a_{dt} \\ lev_t &= \frac{M_{t,t+1} \nu_{t+1} \left(\frac{r_t}{\pi_{t+1}} + (r_{g,t+1} - r_{b,t+1}) lev_{g,t} - \frac{\kappa_F}{2} \left(\frac{lev_{g,t}}{lev_t} - b^* \right)^2 \right)}{\theta - M_{t,t+1} \nu_{t+1} \left(r_{b,t+1} - \frac{r_t}{\pi_{t+1}} \right)} \end{aligned}$$

Finally, using the previous conditions, we get the resulting formula for ν_t :

$$\nu_t = 1 - \chi + E_t M_{t,t+1} \nu_{t+1} \chi \left(\frac{r_t}{\pi_{t+1}} + (r_{g,t+1} - r_{b,t+1}) lev_{g,t} + lev_t \left(r_{b,t+1} - \frac{r_t}{\pi_{t+1}} \right) - \frac{\kappa_F}{2} \left(\frac{lev_{g,t}}{lev_t} - b^* \right)^2 \right)$$